A Numerical Study on Rate-induced Strength Enhancement in Sand

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Abstract. Rate-dependent shearing response of sand has been of major concern for several geotechnical applications related to dynamic compaction, mine blasts, pile driving and rapid load testing of piles etc. In this regard, transient laboratory tests indicate that the mechanical behaviour of sand may vary significantly when the applied strain rate ranges from 10^{-5}/s to 1/s or even higher. Such rate-dependent response of sand depends on the employed strain rate range along with sand morphology, stress and density state. In the present study, a recently proposed elastic-visco-plastic model by Mukherjee et al. [7] has been adopted for the assessment of rate-dependent mechanical behaviour of three different types of sand i.e., crushed coral, silica and Toyoura sand. The predicted rate-dependent shearing response has been noticed to be in close agreement with the experimental observations from literature and the employed model aptly captures the rate effects in different sand types by altering the appropriate model parameters. All the three sand specimens have been subjected to the drained triaxial shearing at a same density and confining pressure, and the rate-induced strength enhancement has been assessed in terms of peak shear stress, peak friction angle and axial strain at peak.

Keywords: Strain rate effect, Visco-plastic model, Drained shearing response, Triaxial test, Sand.

1 Introduction

The shearing response of sand under varying strain rate has been of significance for many military and civilian applications such as blast resistant infrastructure, ground improvement, rapid pile penetration and pile load testing etc. The strain rate range associated with these applications are in the range of 10^{-2}/s or higher [3, 4] and the mechanical behavior of sand has been noticed to vary significantly for such intermediate to high strain rate range [13]. Rate effects generally result in enhanced peak shear strength and stiffness [15, 17]. The strength mobilization mechanism behind this observed enhancement involves inertial and viscous effects, suppressed particle rearrangement and subsequent volume change tendencies, and reduced particle crushing at higher confinements [9]. Further, the rate-dependent response of sand has been observed to vary significantly with density and stress state along with sand morphology i.e., mineralogy, shape, size and gradation [14]. Rate effects are more pronounced in dense sand than in loose ones due to its higher tendency to dilate. Also, rate effects in loose sand are evident at lower confinements owing to the significant role of particle rearrangement, inertial and viscous effects; whereas, rate sensitivity is more significant for dense sands at higher confinement due to associated particle crushing phenomena [5, 18]. Well gradation and angularity in sand particles result in higher interlocking and thus, exhibits higher resistance to particle rearrangement as compared to uniform gradation and rounded sand particles. As a result, well graded angular sands are more susceptible to strain rate changes [2, 6]. Also, at higher confinement, well-graded aggregates exhibit less compressive response and are therefore less prone to grain crushing with increasing strain rate as compared to uniformly graded aggregates [12].

Addressing the rate-dependent mechanical behavior of sand in numerical modelling becomes imperative for the design of geotechnical applications involved in infrastructural development. In this regard, a robust constitutive model needs to be employed to study the rate effect in sand under varying density, confinement and sand morphology. Generally, the rate-dependent response of soil is modelled within elastic-visco-plastic framework such as use of overstress type models. Following this approach, Mukherjee et al. [7] had proposed a 3D non associative critical state model comprising of both shear and volumetric hardening, and incorporating the mathematical framework proposed by Perzyna [10, 11]. The proposed model was employed to investigate the strain localization phenomenon in drained biaxial test and recently, it has been used to study the strain rate effect on mechanical behavior of Toyoura sand subjected to varying density and confinement under drained triaxial loading condition [8]. It has been found that the proposed model can suitably capture some of the key features of the rate-dependent mechanical behavior of sand, such as distinct peaks with enhanced shear strength followed by post-peak softening and suppressed compression.

In the present study, the rate-dependent constitutive model proposed by Mukherjee et al. [7] has been adopted for the assessment of shearing response in sands with different morphology i.e., silica sand, Toyoura sand and crushed coral sand . Each of the sand types has been assessed considering same relative density ($RD = 60\%$) and initial confinement ($p_0' = 100$ kPa) when subjected to intermediate strain rate range under drained triaxial loading condition. The constitutive model parameters have been calibrated first based on the experimental data of drained triaxial test reported in the existing literature and then the predicted shearing response has been validated against the experimental observation corresponding to each of the sand types. The rate-dependent shearing response has been further assessed in terms of enhancement in peak shear strength, peak friction angle, and axial strain at peak.

2 Modelling aspects for capturing rate-dependent shearing response of sand

2.1 Employed rate-dependent constitutive model and calibrated parameters for the selected sand types

The recently proposed elastic-visco-plastic model by Mukherjee et al. [7] has been employed here to simulate the rate-dependent drained shearing response of different types of sands, namely, silica sand, Toyoura sand and crushed coral sand. The properties of each of these sand types are given in Table 1. The employed elastic-visco-plastic model is a critical state based overstress type model incorporating Perzyna-type mathematical framework [10]. The shearing response can be captured through the incremental constitutive relation formulated within a small deformation framework as follows

$$\Delta\sigma'_{ab} = C_{abcd}(\Delta\varepsilon_{cd} - \Delta\varepsilon^{vp}_{cd}) \quad (1)$$

where $\Delta\sigma'_{ab}$, $\Delta\varepsilon_{cd}$, $\Delta\varepsilon^{vp}_{cd}$ represents the incremental effective stress, total strain, visco-plastic strain tensor, respectively and C_{abcd} is the elastic stiffness tensor. Visco-plastic strain evolution is governed by the Perzyna-type flow rule with two yield surfaces i.e., static and dynamic, and a plastic potential surface (G_p), which is specified by the following equation

$$\dot{\varepsilon}^{vp}_{ab} = \dot{\varepsilon}_{ref}\left(\frac{\eta_{yd}}{\eta_{ys}}\right)^n \frac{\partial G_p}{\partial \sigma_{ab}}, \text{ when } \frac{\eta_{yd}}{\eta_{ys}} \geq 1 \quad (2)$$

where, , η_{yd} and η_{ys} denotes the dynamic and static shear stress ratios, $\dot{\varepsilon}_{ref}$ is the reference axial strain rate and n is the power law exponent controlling the rate-dependent behavior. The dynamic yield surface evolves with the evolution in visco-plastic strain.

Table 1. Properties of different types of sands [14, 17, 19]

Sand type	Silica sand	Toyoura sand	Crushed coral sand
Soil classification	SP	SP	SP
Mineralogy	Silica	Quartz	Calcareous
Specific gravity	2.66	2.645	2.86
Average particle size, D_{50} (mm)	0.34	0.19	0.32
Uniformity coefficient (C_u)	1.59	1.56	2.18
Maximum void ratio (e_{max})	0.86	0.99	1.20
Minimum void ratio (e_{min})	0.44	0.62	0.74
Initial void ratio (e_0) at RD=60%	0.61	0.77	0.92
Particle shape	Sub rounded to sub angular	Sub angular to angular	Angular

The calibration of model parameters corresponding to each of the sand types have been carried out through numerical simulations. In this regard, MATLAB code has been developed based on strain controlled fully explicit stress update algorithm given by Wang et al. [16] for simulating the rate-dependent drained triaxial test employing the elastic-visco-plastic model of Mukherjee et al. [7]. The calibrated model parameters for each of the sand types are given in Table 2. Here, ν is the poisson's ratio, G represents shear modulus, M_{cr} is the critical state stress ratio, Λ is the slope of critical state line in the compression plane, Γ is the specific volume corresponding to mean effective stress of 1 kPa, a represents the visco-plastic shear strain required to reach 50% of the static peak shear stress ratio, k is a model parameter linking peak shear stress ratio and state variable, n is the power law exponent and $\dot{\varepsilon}_{ref}$ is the reference axial strain rate.

Table 2. Calibrated model parameters for different types of sands.

Sand type	ν	G (kPa)	M_{cr}	Λ	Γ	a	k	n	$\dot{\varepsilon}_{ref}$
Silica sand	0.3	$G_0/3$	1.1	0.025	1.94	0.006	3	28	10^{-4}/s
Toyoura sand	0.3	$G_0/12$	1.15	0.025	2.0	0.0001	2	45	5×10^{-5}/s
Crushed coral sand	0.3	$G_0/3$	1.4	0.036	2.25	0.0016	2.8	42	2.3×10^{-3}/s

** G_0 represents small strain shear modulus which is function of specific volume and mean effective stress.

2.2 Validation of the predicted rate-dependent shearing response

Rate-dependent drained triaxial simulations have been carried out further with the developed MATLAB code employing the referred elastic-visco-plastic constitutive model and the calibrated parameter sets. The simulated stress-strain responses for the different sand types have been plotted in Fig. 1 along with the experimental results reported in literature corresponding to varying strain rate range [14, 17, 19]. It can be noticed that the employed model suitably captures the rate induced enhancement in stiffness and strength along with a generic trend of distinct peak followed by a softening response at higher strain rates in case of the dense sand.

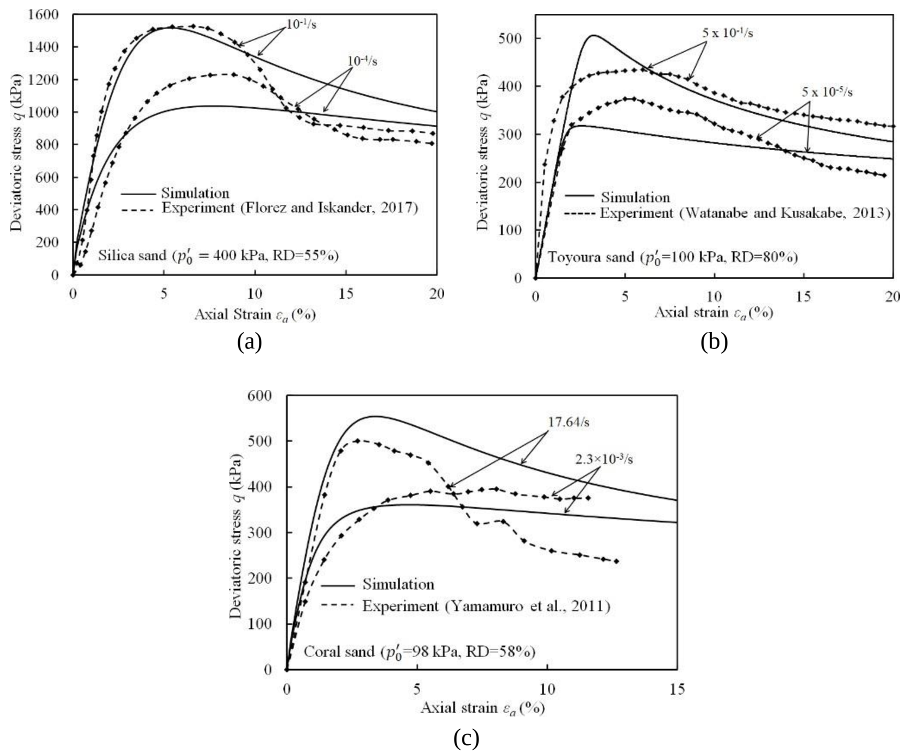


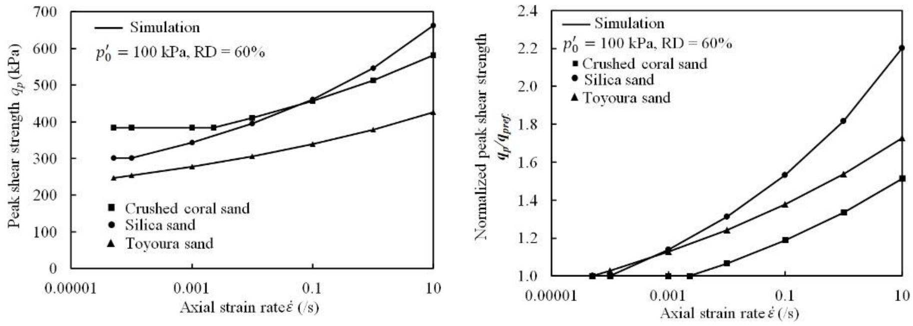
Fig. 1 Evolution of stress-strain response for (a) silica sand (b) Toyoura sand and (c) coral sand subjected to rate-dependent drained triaxial condition with varying strain rate.

3 Drained rate-dependent shearing response subjected to triaxial loading condition

The drained shearing response of the three selected sand types has been further assessed under triaxial loading condition with varying strain rate employing the developed MATLAB code and incorporating the calibrated model parameters in the simulation. For this purpose, same relative density and initial confinement ($p'_0 = 100$ kPa) have been adopted along with a strain rate range of $5x10^{-5}$ /s to 10 /s. The types of sand considered, namely, silica sand, Toyoura sand and crushed coral sand at relative density $RD = 60\%$, which corresponds to initial void ratio values (e_0) of 0.61, 0.77 and 0.92, respectively. Based on the calibrated critical state parameters, the critical state void ratios (e_c) for the these three types of sands i.e., silica sand, Toyoura sand and crushed coral sand at $p'_0 = 100$ kPa have been estimated to be 0.825, 0.885 and 1.08, respectively. Hence, the considered initial void ratios $e_0 = 0.61$, 0.77 and 0.92 corresponding to $RD = 60\%$ represents a dense state of sand. In the following sections, the assessment of the rate-dependent shearing response has been carried out in terms of peak shear strength (q_p), peak friction angle (Φ_p) and axial strain at peak (ε_p).

3.1 Influence on peak shear strength

The variation in peak shear strength with increasing strain rate for different types of sands has been plotted in Fig. 2(a) along with normalized peak shear strength response in Fig. 2(b). It can be noticed that the crushed coral sand yields a higher peak shear strength than Toyoura sand owing to the angularity of particles and a higher uniformity coefficient which represents better gradation. As a result, the crushed coral sand exhibits better interlocking resulting in higher resistance to particle rearrangement. The rate induced enhancement in peak shear strength is comparable for both the crushed coral and Toyoura sand due to similar strain rate sensitivities as indicated by power law exponent n value reported in Table 2. Silica sand exhibits higher peak shear strength than Toyoura sand and the overall increment in peak shear strength is also highest for silica sand due to higher strain rate sensitivity ($n = 28$). Further, in comparison to the crushed coral sand, silica sand exhibits a lower peak shear strength up to a strain rate 10^{-1}/s and then attains a higher peak shear strength . It can be noticed from Fig. 2 that the rate effect becomes imperative only after the reference strain rate for each of the sand types. Below the reference strain rate, the value of peak shear strength remains constant irrespective of the change in the strain rate. The overall rate induced enhancement in the peak shear strength beyond the respective reference strain rates has been noticed to be around 120% for silica sand, 73% for Toyoura sand and 50% for crushed coral sand. It can be noticed from these increments that lower value of power law exponent (n) and reference strain rate ($\dot{\varepsilon}_{ref}$) represents a higher strain rate sensitivity.

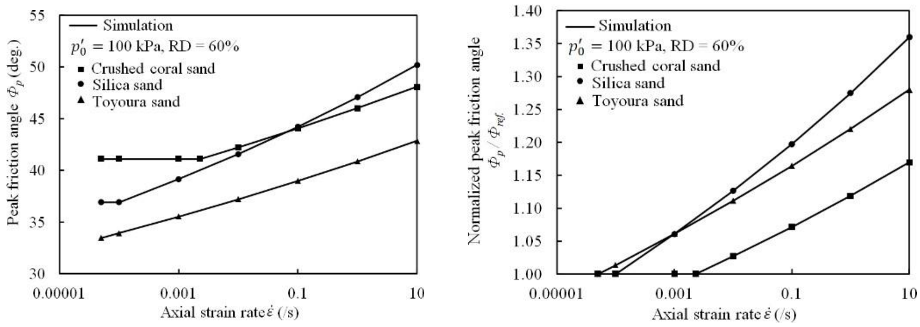


(a) (b)

Fig. 2 Variation in the (a) peak shear strength (q_p) and (b) normalized peak shear strength ($q_p/q_{p_{ref}}$) for different sand types over the selected strain rate range when subjected to drained triaxial condition at initial confinement $p'_0 = 100$ kPa and relative density $RD = 60\%$.

3.2 Influence on peak friction angle

The variation in peak friction angle with increasing strain rate for different types of sands has been plotted in Fig. 3(a) along with the normalized peak friction angle response in Fig. 3(b). The increment in the peak friction angle corresponding to the enhancement in peak shear strength has been noticed to be 36% for silica sand, 28% for Toyoura sand and 17% for crushed coral sand (Fig. 3b). It can be observed that small variation in the values of peak friction angle with varying strain rate results in a marked variation in the peak shear strength values.

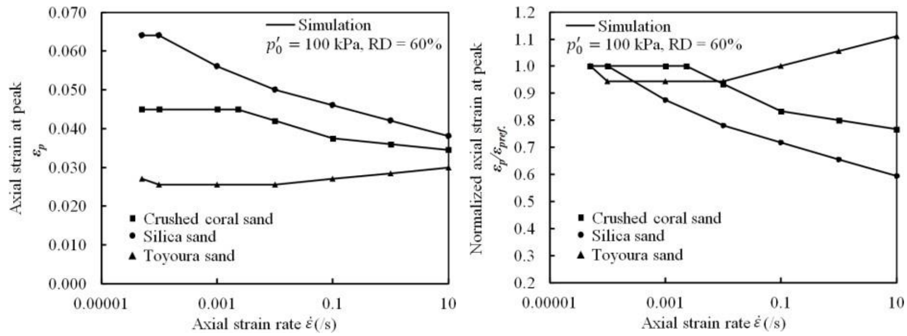


(a) (b)

Fig. 3 Variation in the (a) peak friction angle (Φ_p) and (b) normalized friction angle ($\Phi_p/\Phi_{p_{ref}}$) for different sand types over the selected strain rate range when subjected to drained triaxial condition at initial confinement $p'_0 = 100$ kPa and relative density $RD = 60\%$.

3.3 Influence on axial strain at peak shear stress

The influence of rate effect on axial strains at peak shear stress (ε_p) has been plotted in Fig. 4 along with the normalized axial strain ($\varepsilon_p/\varepsilon_{p_{ref}}$) response. It can be noticed that ε_p values decrease with increasing strain rate in case of silica and crushed coral sand, which indicates an early onset of peak. The variation of axial strain at onset of peak for silica sand has been noticed to be higher than that in crushed coral sand, which is attributed to the higher strain rate sensitivity of silica sand. In case of Toyoura sand, a slight delayed onset of peak has been noticed at strain rate above 10^{-2}/s.



(a) (b)

Fig. 4 Variation in the (a) axial strain at peak (ε_p) and (b) normalized axial strain at peak ($\varepsilon_p/\varepsilon_{p_{ref}}$) for different sand types over the selected strain rate range when subjected to drained triaxial condition at initial confinement $p'_0 = 100$ kPa and relative density $RD = 60\%$.

4 Conclusion

In the present study, the rate-dependent shearing response of three sands, namely, silica, Toyoura and crushed coral sand having different morphology has been analyzed corresponding to a specific relative density and initial confinement over the intermediate strain rate range of $5\text{x}10^{-5}$ /s to 10 /s. The proposed study attempts to explore the influence of sand morphology on the strain rate effect. In this regard, a recently proposed rate-dependent elastic-visco-plastic constitutive model has been adopted. A MATLAB code has been written based on fully explicit stress update algorithm in order to simulate the drained triaxial test subjected to varying strain rate. Accordingly, the model parameters are calibrated for these different sand types based on the experimental data of drained triaxial test available in the existing literature.

The influence of varying strain rate on the peak shear strength, peak friction angle and axial strain at peak has been assessed from the triaxial test simulations of these different sand types at a relative density of 60% and an initial confining pressure of 100 kPa. Beyond the reference strain rate, the overall enhancement in the peak shear strength over the considered strain rate regime has been noticed to be around 120%, 73% and 50% for silica sand, Toyoura sand and crushed coral sand respectively. The subsequent increase in the peak friction angle has been observed to be 36%, 28% and 17%, respectively. Further, the variation of axial strain at peak indicates an early onset of peak for silica and crushed coral sand; whereas, a slight delayed onset of peak has been noticed in case of Toyoura sand. It can be concluded that, with proper calibration of model parameters the employed model can aptly capture the influence of sand morphology on the strain rate effect.

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