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Active Learning Framework for Reliability Estimation of Rock Slopes

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Abstract. Response surfaces are commonly adopted as a surrogate for complex performance functions due to their ability to solve rock slope reliability problems with low computational costs. Most widely used methods employ a static scheme that uses input (rock properties) and output (factor of safety) samples generated using some experimental design to obtain the best fit parameters of the response surface. However, since the selection of input samples is not optimized, the increase in accuracy of the response surface often comes at a cost of an increased number of performance function evaluations, particularly, for slopes having a low probability of failure (P_f). This is addressed by an active learning scheme that iteratively selects input samples that improve the prediction of the response surface around the failure region. In this paper, active learning scheme with support vector machine (SVM) is adopted for estimating the P_f a rock slope along Rishikesh – Badrinath highway against planar failure. The analytical expression for the factor of safety is utilized for conducting Monte Carlo simulation to estimate P_f, which is treated as a benchmark for determining the accuracy of the proposed method. Comparison with static scheme SVM illustrates the advantages of active learning scheme in increasing the accuracy in estimating the P_f for a similar number of performance function evaluations.

Keywords: Active learning, Support Vector Scheme, Rock slope planar failure, Probability of failure

1 Introduction

Analysis of the stability of rock slopes is one of the most important problems in rock engineering. Rock slope failures can either be structurally controlled in which sliding of rock mass occurs along the discontinuities or stress-controlled which is characterized by shear failure of the rock mass under self-weight. The traditional design approach consists of deterministic characterization of intact rock and rock mass properties followed by estimation of factor of safety (FOS) of the slope. Several limit equilibrium methods (LEM) and numerical techniques have been developed and applied for FOS estimation (Hoek and Bray, 1981; Kanungo et al., 2013; Tiwari and Latha, 2015; Pandit, 2018). If the FOS of the slope exceeds a certain value it is considered safe. However, since the rock mass is a geo-material, its parameters are subjected to uncertainty which leads to uncertainty in the estimated FOS.

This problem can be addressed by probabilistic stability assessment that considers the uncertainty in the rock mass parameters and estimates the probability of failure (P_f) of the slope, which must be below some threshold value for it to be considered safe. Several probabilistic techniques such as first order reliability method (FORM), point estimate method (PEM) and Monte Carlo simulations (MCS) have been utilized in conjunction with LEMs and numerical methods to estimate P_f (Ang and Tang, 1975). MCS is a widely used method for P_f estimation that involves the evaluation of the performance function (LEM or other numerical methods), corresponding to a larger number of randomly realized rock mass parameters. This becomes computationally challenging if the performance function requires large run times. This can be addressed by constructing a response surface, that acts as a simpler surrogate function relating rock mass parameters and FOS. This function can be a polynomial, Radial basis function (RBF), Gaussian process model, Support vector regression (SVR), etc. (Roy and Chakraborty, 2020).

This approximation of the performance function is achieved by training the surrogate function using the training data points chosen according to the design of experiments (DOE). Space filling DOE like Latin hypercube sampling (LHS) is widely adopted. However, the accuracy of the response surface in approximating the true P_f of the slope depends on its accuracy near the failure surface. This is difficult to achieve with static DOE samples. Several researchers have addressed this problem by enriching DOE iteratively by taking data points close to the approximated failure surface (Echard et al., 2011; Roy and Chakraborty, 2020). However, limited research has been done on the applicability of adaptive response surface methods on P_f estimation for rock slopes.

In this paper, P_f of a rock slope located along Rishikesh – Badrinath highway susceptible to planar failure is estimated using direct MCS and SVR based response surface with both static and adaptive strategies. Adaptive strategy is executed in two stages – First, an initial LHS DOE is used to approximate the performance function and second - new data points are added iteratively to the initial DOE which are nearest to the failure surface as approximated by the previous DOE and located as far as possible. This sequential addition is performed until convergence is achieved. It is demonstrated that P_f estimate of the rock slope is greatly improved for a similar number of performance function evaluations when compared with the static strategy.

2 Support Vector Regression

The SVR is widely used to learn the linear/non-linear relationship between the input and the output features. It is obtained by minimizing a ε-insensitive loss function which penalizes the points only when they are located beyond the tube enclosed by ε (non-negative precision tolerance) (Drucker et al., 1996).

Let $\boldsymbol{X}$ be the vector of input variables with output y. For a given training data set $\{\boldsymbol{x_1}, \boldsymbol{x_2}, \dots \boldsymbol{x_n}\}$, $\boldsymbol{x_i} \in \mathbb{R}^m$ and corresponding output $\{y_1, y_2, \dots, y_n\}, y \in \mathbb{R}$, the support vectors are obtained by solving the optimization problem –

Maximize

$$w(\alpha, \alpha^*) = \frac{-1}{2} \sum_{i,j=1}^{m} (\alpha_i - \alpha_i^*)(\alpha_j - \alpha_j^*) K(x_i, x_j) + \varepsilon \sum_{i=1}^{m} y_i (\alpha_i - \alpha_i^*) - \sum_{i=1}^{m} (\alpha_i - \alpha_i^*) \qquad (1)$$

Subjected to

$$\begin{cases} \sum_{i=1}^{m} (\alpha_i - \alpha_i^*) = 0 & 0 \le \alpha_i \\ \alpha_i \le C & i = 1,2, \dots . m \end{cases} \qquad (2)$$

where C is regularization constant, α_i, α_i^* are Lagrangian multipliers, and $K(\boldsymbol{x_i}, \boldsymbol{x})$ is the RBF kernel function. After solving the optimization problem, SVR can be expressed as

$$y = f(\boldsymbol{x}) = \sum_{i=1}^{n} (\alpha_i - \alpha_i^*) K(\boldsymbol{x_i}, \boldsymbol{x}) + b \qquad (3)$$

where b is an offset parameter.

The accuracy of SVR is generally estimated by comparing the SVR output and actual output of the test data set and evaluating the goodness-of-fit measure (such as RMSE, coefficient of determination R^2, etc.).

2.1 Static SVR

Initially, a predefined number of input data sets are obtained using some design of experiments (space filling designs such as LHS), which are input into the performance function to get corresponding outputs. This input-output pair called the training data set, is utilized to construct a SVR. A separate test input-output pair is then utilized to estimate the accuracy of the SVR. If accuracy is low, then the number of input data set is increased and the steps are repeated till the desired accuracy is achieved. However, higher goodness-of-fit doesn't guarantee higher accuracy in the estimation of P_f, since the location of the test data set might be away from the surface of the input domain where FOS is 1 (limit state).

2.2 Adaptive SVR

Adaptive SVR aims to select such input data sets that significantly improve the accuracy of SVR, in case desired accuracy is not achieved. Since the objective is to find P_f of the rock slope, the accuracy of SVR should be high in the region around the limit state.

Initially, static SVR - $SVR^{(0)}$ is constructed with small training data sets, resulting in lower accuracy. Now, the next set of training data points are chosen such that they are located near the limit state as predicted by $SVR^{(0)}$ and located far from other training data sets (to ensure space filling). Subsequently, using the newly selected data sets, a new SVR - $SVR^{(1)}$ is constructed, having more accuracy than $SVR^{(0)}$. Similar, addition of new input data sets is performed k times to get a final SVR - $SVR^{(k)}$, beyond which no significant improvement is possible.

3 Description of the rock slope

A rock slope located along the Rishikesh – Badrinath highway in Uttarakhand, India is considered (Pain, 2012; Kumar and Tiwari, 2022; Pandit et. al, 2023). The major rock type of the slope is Quartzite with a unit weight of 26 kN/m^3. Barton-Bandis strength model is assumed to represent the resistance offered by the rock joint against the driving force (Fig. 1). Table 1 consists of the estimated properties and their statistical distributions estimated using ISRM methods. The FOS of the slope is obtained as resistive shear force along the discontinuity divided by the driving force using the equation given below (for derivation refer Kumar and Tiwari (2022)):

$$FOS = \frac{\left[W\{(1-k_v)\cos(\theta_p) - k_h \sin(\theta_p)\} - \left(\frac{1}{2}\gamma_w {Z_w}^2\right)\sin(\theta_p) - \left\{\frac{1}{2}\gamma_w Z_w \times (H-Z)cosec(\theta_p)\right\}\right] \times tan\left[\phi_r + JRC \times log\left(\frac{JCS}{\sigma_n}\right)\right]}{\left[W\{(1-k_v)\sin(\theta_p) + k_h \cos(\theta_p)\} + \left(\frac{1}{2}\gamma_w {Z_w}^2\right)\cos(\theta_p)\right]} \tag{4}$$

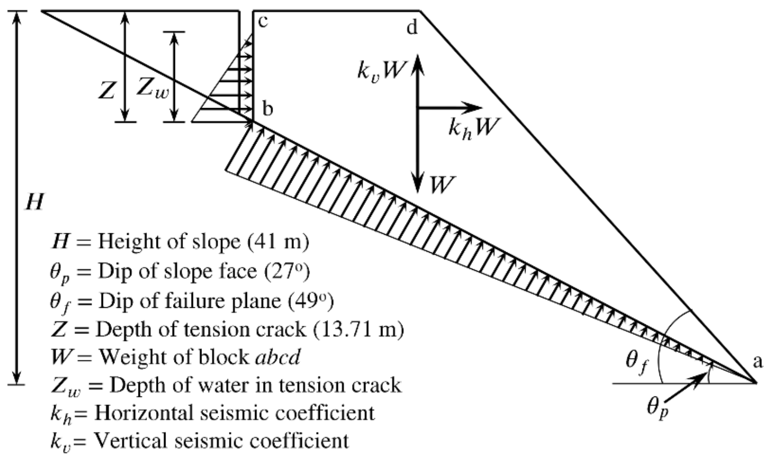

Fig. 1: Rock slope geometry (Kumar and Tiwari, 2022)

The performance function (Eq. 4) is coded in the Python and the deterministic FOS obtained is 1.12 when evaluated at mean values of input parameters, which is equal to the FOS obtained in Kumar and Tiwari (2022).

Table 1. Rock joint and other properties along with their probability distribution (Pandit et. al, 2023).

Variable	Mean	SD	PD
Joint Roughness Coefficient (JRC)	3.63	1.0903	Uniform
Residual friction angle (ϕ_r) (°)	31.96	2.1873	Lognormal
Joint wall Compressive Strength (JCS) (MPa)	10.18	2.5538	Lognormal
Depth of water in tension crack (Z_w) (m)	6.86	–	Truncated Exponential
Horizontal seismic coefficient (k_h)	0.12	–	Truncated Exponential
Vertical seismic coefficient (k_v)	0.08	–	Truncated Exponential

4 Estimation of P_f using different methods

The estimation of P_f using analytical performance function and approximated different SVRs is performed in Python 3.0.

4.1 Monte Carlo simulation

P_f of 0.27 for the rock slope is obtained by conducting 10^5 MC simulations on Eq. 4. This ensures sufficient convergence of the P_f estimate with coefficient of variation of 0.5 %. Since, this is exactly known performance function, the P_f thus obtained is treated as reference (P_f^{ref}) when comparing the P_f obtained from static and adaptive SVR approaches.

4.2 Static SVR

In this approach, Eq. 4 is approximated by SVR using training data sets obtained from LHS by static approach. 51 LHS samples are utilized to construct the SVR and 25 test data sets are utilized to evaluate the SVR's goodness-of-fit measures - R^2 value and sum of squared error. P_f of the rock slope is estimated by conducting 10^5 MC simulations on the static SVR.

Since LHS is a type of random sampling, the above procedure is repeated by changing the seed of the random number generator. P_f obtained from each SVR is denoted as $P_f^{S_1}$, where subscript S_1 refers to static SVR with seed value 1. Fig. 2 shows values of goodness-of-fit and the absolute difference $|P_f^{ref} - P_f^{S_i}|$ obtained for 7 different seeds. It can be seen that the R^2 values range from 0.85 to 0.77, while the $|P_f^{ref} - P_f^{S_i}|$ range from 0.09 to 0.14.

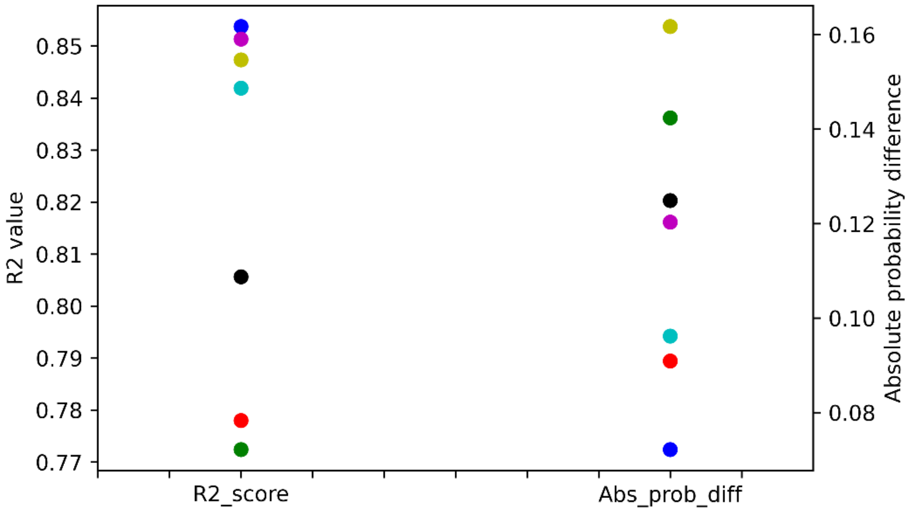

Fig. 2: R^2 value (Left y-axis) and $|P_f^{ref} - P_f^{S_i}|$ (Right y-axis) for different seeds of random number generator

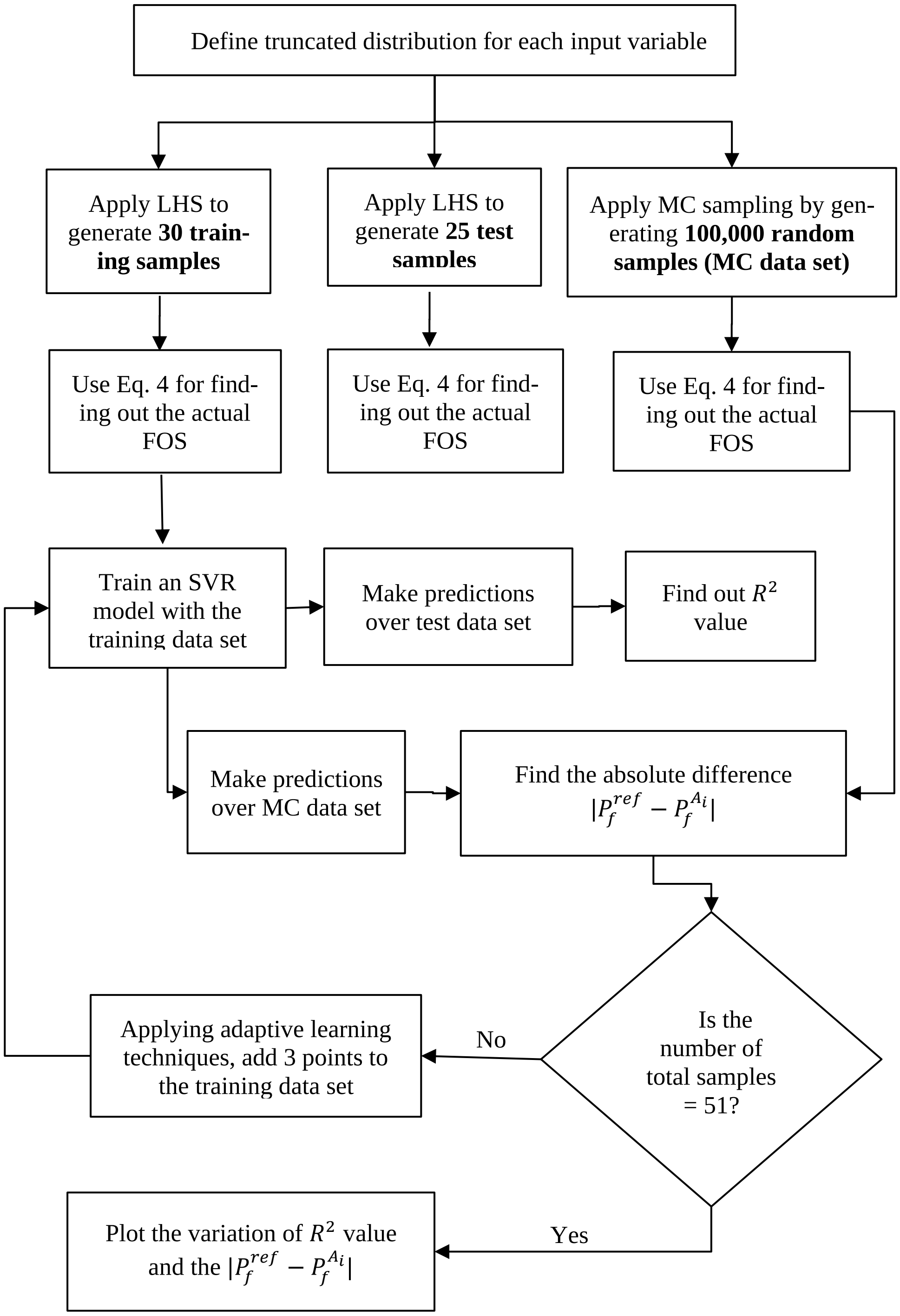

Fig. 3: Flowchart indicating the procedure adopted for adaptive SVR

4.3 Adaptive SVR

The procedure adopted for adaptive SVR is provided as a flowchart in Fig. 3. For adaptive SVR, $SVR^{(0)}$ (refer section 2.2) is obtained initially with 30 LHS samples. Afterwards, 10^5 MC samples are evaluated on $SVR^{(0)}$ and those points are grouped in a candidate pool that leads to FOS values within the range [0.9, 1.1]. This ensures that all candidate points lie close to the limit state as predicted by $SVR^{(0)}$. Next, all candidate points are sorted according to the decreasing average Euclidean distance from all the training data points. Finally, the first three points are selected as adaptive addition to the initial training points, ensuring adaptive points are located as far as possible from training points satisfying the space filling requirements. Afterwards, updated training data set is utilized to construct $SVR^{(1)}$ which is more accurate than $SVR^{(0)}$. This sequential addition of adaptively selected points and improvement in SVR is performed for 7 iterations to arrive at $SVR^{(7)}$. Similar to static SVR, adaptive SVR is conducted with 7 different random number generators, with P_f obtained denoted as $P_f^{A_i}$. Adaptive improvement with each iteration can be seen in Fig. 4, whereas, Fig. 5 shows the results of the analysis. It can be seen that R^2 values range from 0.88 to 0.76 and $|P_f^{ref} - P_f^{A_i}|$ range from 0.04 to 0.09.

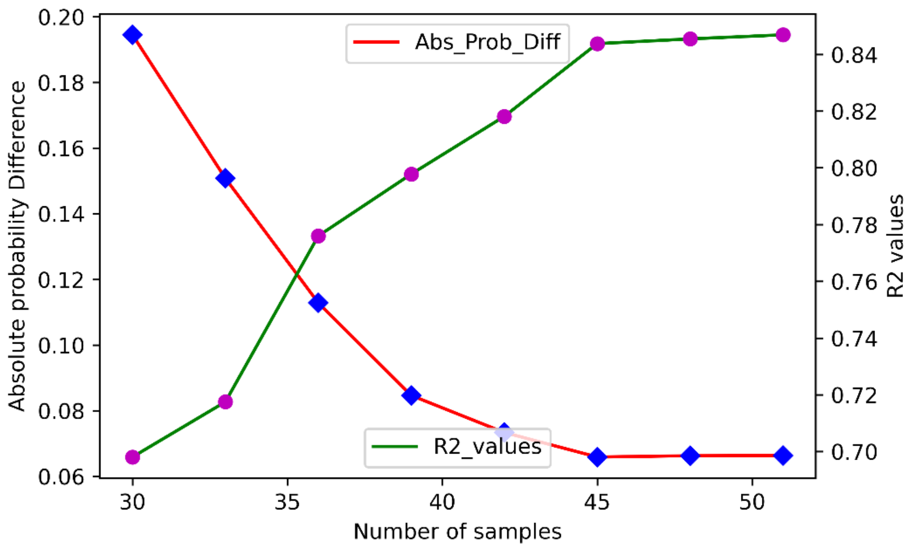

Fig. 4: Adaptive improvement in R^2 value and decrease in $|P_f^{ref} - P_f^{A_i}|$ for a particular seed of random number generator

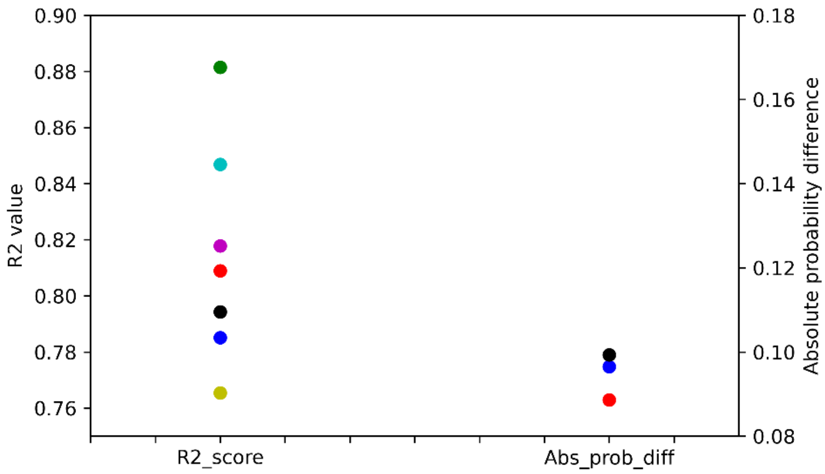

Fig. 5: R^2 value (Left y-axis) and $|P_f^{ref} - P_f^{A_i}|$ (Right y-axis) for different seeds of random number generator

5 Discussions

Adaptive SVR is compared with static SVR against the reference obtained by the known performance function. Static SVR usually employs LHS sampling to obtain training data sets, the accuracy of such SVR depends on the seed of the random number generator as demonstrated in Fig. 2 using R^2 values and $|P_f^{ref} - P_f^{S_i}|$. It is also seen that a better R^2 value does not imply a better prediction of P_f as the test data which is used to estimate R^2 is located randomly and not necessarily near the limit state. This limitation is important since static schemes of the response surfaces are widely adopted in the geotechnical discipline.

Adaptive SVR sequentially improves the accuracy of estimating P_f of the rock slope, as evident in Fig. 6 by observing the reduction in $|P_f^{ref} - P_f^{A_i}|$ with adaptive addition of training data points. For the same number of performance evaluations as static SVR, a benefit defined as $|P_f^{ref} - P_f^{S_i}|$ - $|P_f^{ref} - P_f^{A_i}|$ of up to 0.13 is obtained. For all adaptive SVR, with different values of seed number, final $|P_f^{ref} - P_f^{A_i}|$ is below 0.1. Additionally, the fluctuation is observed in R^2 values as $|P_f^{ref} - P_f^{A_i}|$ is decreasing shows that high R^2 value does not always imply more accuracy in estimating the P_f.

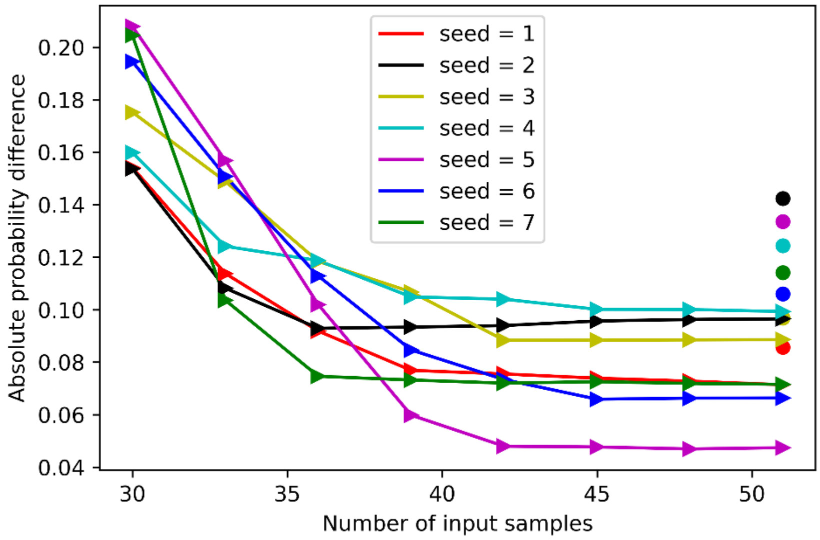

Fig. 6: Adaptive improvement (decrease) in $|P_f^{ref} - P_f^{A_i}|$ (triangles) and comparison with values of $|P_f^{ref} - P_f^{S_i}|$ (circles)

6 Conclusions

In the current work, a probabilistic stability analysis of rock slope located on the Rishikesh - Badrinath highway susceptible to planar failure is performed. The performance function of the slope is exactly known in the form of a limit equilibrium equation which facilitates the comparison of the accuracy of static SVR and adaptive SVR. It can be concluded that adaptive SVR offers a monotonous increase in accuracy in estimating the P_f with the sequential addition of adaptive training data. If enough iterations are performed, the desired accuracy can be achieved. However, as the accuracy of static SVR is defined in terms of goodness-of-fit measures, its high value does not always provide an increase in accuracy in P_f estimation. It is demonstrated in the paper that adaptive SVR provides more accuracy compared to static SVR for a similar number of performance function evaluations.

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