Kochi Chapter

Indian Geotechnical Conference
IGC 2022
15th – 17th December, 2022, Kochi

Comparative analysis of settlement and efficiencies of pile groups with different configurations using FEM

Siddhartha Mukherjee[1] and Asheequl Irshad[1]

[1] Techno International New Town, Kolkata, India-700156
`Siddhartha.mukherjee@tict.edu.in`
`asheequlirshad@gmail.com`

Abstract: This study intends to investigate the settlement response of different configurations of group of piles subjected to compressive load. Using Plaxis 3D, large diameter cylindrical concrete piles were modelled as statically loaded single and in groups and numerical analysis were carried out. Load-settlement curves obtained from numerical analysis are utilized to estimate the efficiency coefficient of pile groups. Settlement response of the single pile from numerical analysis is validated with field test results obtained from published literature. Sensitivity analysis carried out with different mesh sizes for optimization of results. The study indicates the decrease in efficiency coefficient with the increase in number of piles in group due to overlapping of stresses. Analysis of concentrically loaded single row pile groups shows that individual contribution of piles to the total group resistance decreases with increasing distance from the center of the group. Pertinence of the efficiency coefficient obtained from numerical analysis is assessed by comparing the results with those calculated from the well-known formulas available in public sources. Furthermore, results from analysis of selective pile group with Hardening Soil Model shows marginally higher capacity as compared to Mohr-Coulomb model particularly for higher settlements.

Keywords: Plaxis 3D; Pile Group; Efficiency coefficient; Numerical analysis

1 Introduction

Deep foundation in form of a collection of piles connected at the top to a pile cap is frequently necessary for heavily loaded structures on soft soil. It is realistic to anticipate that soil pressures caused by end bearing or side friction will overlap when multiple piles are grouped together. It is a matter of concern to estimate or predict the load carrying capacity of the pile group (Q_g) as compared to the summation of the individual capacity (Q_s) of all piles in group. Interaction effect between piles in the group is estimated by the group efficiency coefficient (C_g) which is defined as the ratio of Q_g to n times Q_s where n is the number of piles in the group. Some researchers have tried to estimate efficiency of pile group from experimental results at laboratory scale (Barden and Monckton 1970 [1]; Briaud *et al*. 1989 [2]). Majority of previous methods involve idealizing or simplifying soil profile in order to make calculations easier, sacrificing accuracy and neglecting the requirement of taking response of pile group and its variability with level of settlement into account. Therefore, the question remains whether existing formulae for estimation of pile group efficiency are still applicable for real-life field condition. Modern computing devices and powerful computer codes have made it easier to model the pile-soil system as a single composite continuum and predict the mass response of different soil layers and their interactions with piles using numerical analysis utilizing Finite Element Method (FEM). Previous FEM analysis (Comodromos 2004 [3]) showed significant interactions between piles within groups. However, most of the previous pile group response studies using FEM considered only some regular (rectangle or square) arrangements of piles in group. Pile group response under static load for different geometrical configurations other than rectangle or square shapes has not studied yet. Hence, to understand interaction between piles in pile group and its overall settlement response under static load for different geometrical shapes, full scale study for each configurations needs to be done. Present study aims to address this issue by conducting full scale numerical analysis of pile groups of various configurations to obtain the settlement responses under static load and thereby assess efficiency of pile groups on basis of load-settlement response. The obtained efficiencies from numerical study are then compared with efficiency calculated by well-known formulas available in literature to check their relevance in predicting pile-group response.

2 Numerical study

2.1 Finite Element Modelling

Geometry: Soil profile and concrete piles have been modelled using the data documented by Naveen et al. 2011 [4]. Plaxis 3D model contains two layers of soil i.e., clay (up to 6 m) followed by soft weathered rock (6m to 20m). Diameter of pile (D) is 1.2 m and length is 15 m. For modelling group piles, soil contour proposed by Naveen et al. [4], was considered insufficient which led to poor and inaccurate results and thus dimension of the model was increased to 50 m x 50 m x 40 m (x, y and z direction). The comparative analysis considers soil as Mohr-Coulomb (MCM) as well as Hardening Soil model (HSM) whereas the pile and pile cap are considered as linear-elastic (LEM). Summary of the property of soil, pile and pile-cap are given in Table 1.

Table 1. Soil and pile parameters used in PLAXIS models

	Clay		Soft weathered rock		Pile and Pile-cap (concrete)
	MCM	HSM	MCM	HSM	LEM
γ_{sat} (kN/m^3)	21	21	22	22	25
γ_{unsat} (kN/m^3)	21	21	22	22	25
E (kN/m^2)	40.00E3	-	100.00E4		30.00E6
E_{50}^{ref} (kN/m^2)	-	40.00E3	-	100.00E4	-
E_{oed}^{ref} (kN/m^2)	-	40.00E3	-	100.00E4	-
E_{ur}^{ref} (kN/m^2)	-	120.00E3	-	300.00E4	-
v	0.3	-	0.33	-	0.2
Cohesion (c_u)	30	30	50	50	-

Friction angle (φ)	20°	20°	25°	25°	-
m	-	0.7	-	0.5	-
p_{ref} (kN/m^2)	-	30	-	30	-

Boundary conditions, interface elements, discretization and meshing: Bottom boundary is considered as rigid, i.e., both horizontal (u) and vertical displacement (v) are zero. Standard fixities are used at left and right boundaries of model. Side boundaries act like rollers such that $u=0$ and $v\neq0$. Soil-structure interaction is modelled by introducing an elastic-plastic element between the piles and the soil to describe the behavior of interfaces. Interface element properties are linked to strength properties of the soil layers. Main interface parameter is strength reduction factor R_{inter} which is assumed 1.0 as considered by Naveen et al. [4]. Global coarseness parameters are used while generating mesh. Average element size and number of generated tetrahedral elements depend on the global coarseness setting. Global coarseness setting of very fine is considered for modelling pile groups as the maximum number of elements were generated by it and thereby most accurate results are expected. However, single pile models using different mesh sizes are also done to understand the effect of mesh variation on the results. Global scale factor is taken as 1.2 and the minimum element size factor is taken as 0.005. Local coarseness parameters automatically generated by Plaxis is used.

Staged Construction: Stages used in analysis of piles are defined as follows:

(i) Pile construction and assigning interface elements in the soil model to allow for pile-soil slip.
(ii) Excavation of soil to a depth of 0.5 m below the pile heads.
(iii) Installation of pile cap (modelled as ‘plate’ in Plaxis 3D) along the top of pile group. To ensure sufficient rigidity for uniform distribution of load on piles, the depth of pile cap was chosen as 1.5 m [this step was not included while modelling a single pile].
(iv) Applying point load on the C.G. at regular increment to generate load-settlement curves.

Excavation of 0.5 m was incorporated in the model to ensure the pile group behaves as free-standing group. No contact in between the pile cap and the soil was provided and thus, there was no initial transfer of load from the pile cap into the soil. Fig. 1 and 2 illustrate the geometry of the model and the pile-volume interface respectively.

2.2 Methodology

Comparative study of compressive load carrying capacities of pile groups having different number of piles as well as same number of piles in different configurations is attempted and settlement responses subjected to concentric vertical loads are observed. Efficiency coefficient of pile groups is calculated using load-settlement curves obtained from FEM analysis. Some models were also analyzed with HSM in order to study variation in results as compared to MCM. The number of piles in groups were considered varying from 2 to 10 with different geometrical arrangements in groups. The nomenclature (I) in the pile group marking denotes the pile group having linear arrangements of pile whereas (II) denotes different pile arrangement other than linear in pile groups with respect to number of piles. Detailed illustration of the arrangement of the piles in different groups are given in Fig. 3. Spacing between piles (Sp) was assumed as 2.5D in groups.

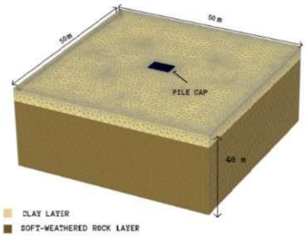


Fig. 1. FEM model of the geometry

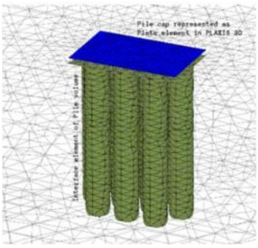


Fig. 2. Pile-volume interface with pile cap

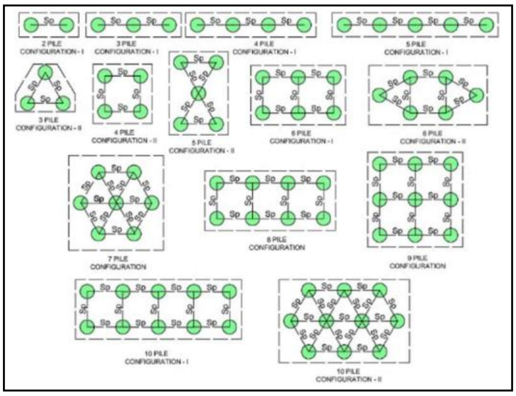


Fig. 3. Arrangement of pile in groups

2.3 Coarseness of mesh

PLAXIS 3D uses 5 basic settings for meshing including very coarse, coarse, medium, fine and very fine. Very fine mesh generates maximum number of nodes and elements and vice-versa. For dynamic analysis, meshing does play a vital role in order to obtain realistic results. However, effect of mesh coarseness is not significant for relatively simple models like this, but is definitely applicable for complex ones. While finer mesh produces more precise and realistic results, it significantly increases execution time whereas a too-coarse mesh will overlook the subtle changes in the stress generated in various regions of the model. (Dey 2011 [5]).

Single pile was modelled and analyzed in all mesh settings and load settlement curves are shown in Fig. 4. It is clearly evident from Fig. 4 that no significant changes in load-settlement curves were recorded due to the mesh size variation. However, minute changes observed in results are shown in magnified view in Fig. 4.

2.4 Validation of Model

Single pile load test data as reported by Naveen et al 2011 [4] is used in this study for validation of model. Vertical settlement corresponding to 8250 kN (final load up to which pile load test was carried out) has been presented in Table 2.

Table 2. Settlement response of PLAXIS models with load test results

Methods	Vertical settlement (mm)
Field test (Naveen et al. 2011)	2.56
PLAXIS 2D Analysis (Naveen et al. 2011)	4.91
PLAXIS 3D Analysis (present study)	4.954

From Fig. 5, it is seen that the load-settlement curve obtained by Plaxis 2D simulation is closed to the field test results up to the settlement value of 2 mm beyond which the pile load test curve showed a different shape. Naveen et al 2011 [4] ascribed this discrepancy to field test issues. Plaxis 3D simulations in present study predict slightly larger settlements during initial loading stage compared to Plaxis 2D curves, but former load-settlement curve follows a similar trend to later ones, yielding nearly identical settlement values at a final load of 8250 kN (see Table 2 and Fig. 5).

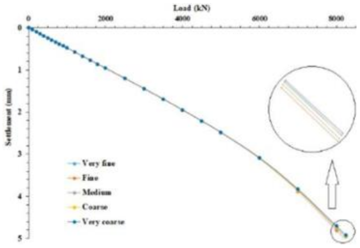


Fig. 4. Effect of mesh coarseness on settlement response for vertically loaded single pile in PLAXIS 3D

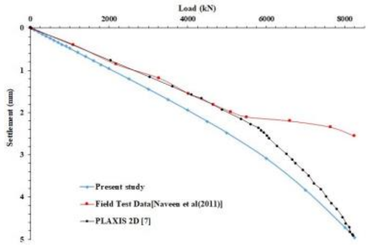


Fig. 5. Validation of load-settlement curve in PLAXIS 3D

3 Results discussion

3.1 Comparison of pile group response against single pile

Fig. 6 illustrates total vertical displacement contour corresponding to a settlement of 25 mm measured at the center point on top of the single pile (Fig. 6a) and pile group (Fig. 6b). From these figures it is evident that lower portion of the pile(s) inside the soft weathered rock layer settles less than the upper part which is situated inside the relatively softer clay layer. Fig. 7 shows relative shear stress contour of soil around the pile for single pile and pile group of 7 piles. Relative shear stress (τ_{rel}) is ratio of mobilized shear stress (τ_{mob}) to the maximum shear stress (τ_{max}), value of which depicts the proximity of each point to failure. A much greater zone of soil around pile has reached near to the failure stress for the case of 7-pile group (Fig. 7a) in comparison with case of single pile (Fig. 7b). Fig. 6 and 7 also illustrate the extent of influence zone with respect to settlement as well as the stress induced in the soil, signifying the requirement of extra depth to model the soil contour. Superposition of stresses occurred because of the action of all piles in the pile groups due to the close spacing (2.5D) of the piles and a larger extent of soil was influenced by group action. This is consistent with the higher load carrying capacity of pile groups where the piles in the group presume to carry load collectively. It is clear from Fig. 6 and 7 that soft weathered rock layer offers more frictional resistance than top clay layer.

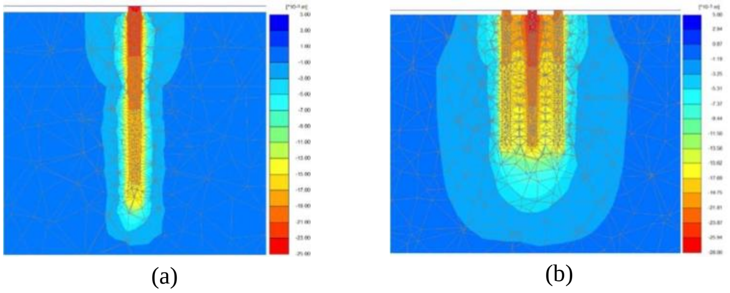

(a) (b)

Fig. 6. Total downward displacement contour corresponding 25 mm settlement obtained from numerical analysis (a) for single pile, (b) for group of 7 piles.

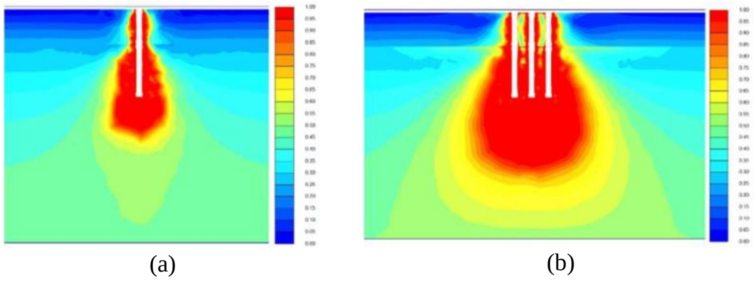
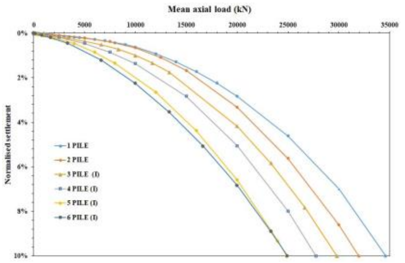

(a) (b)

Fig. 7. Relative shear stress contour obtained from numerical analysis (a) for single pile, (b) for group of 7 piles

Fig. 8 and 9 compare load settlement curve of the single pile with average load settlement curves for pile groups of different geometric configurations. Average load settlement curve for a group of piles is simply the equivalent settlement response of a single pile subjected to an average load, obtained by dividing the total load at each load level by the number of piles in that group. From the curves presented in Fig. 8 and 9 it can be observed that for a given settlement value, average load per pile within a group is less than the load for a single pile. Also, general trend shows that average load capacity of piles in groups decreases as the number of piles increases in a group, except with slight deviation to this rule for pile group marked as ‘5 PILE (II)’ in Fig. 9. The arrangement of the piles in this group may have contributed to this exception. In comparison to pile groups with a more compact arrangement of piles comprising many pile rows, average load-settlement curves for pile groups with single row pile arrangement are distinctly separated as exhibited in Fig. 8.



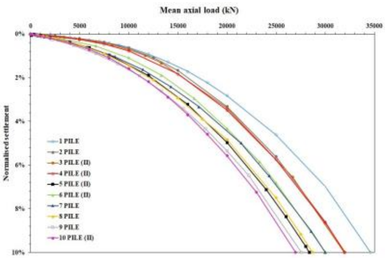
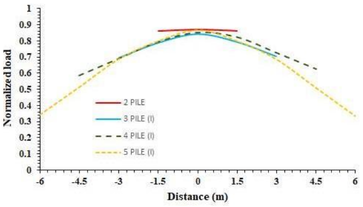


Fig. 8. Load-settlement curve for single pile and average load-settlement curves for pile groups having piles in a single row

Fig. 9. Load-settlement curve for single pile and average load-settlement curves for pile groups

Load distribution of individual piles in group having only one row of piles is presented in Fig. 10. This figure plots the normalized load of individual pile with respect to the distance from the C.G. of pile group. Normalized load of individual pile is defined as ratio of induced axial load in any pile of a group to the load carrying capacity of single pile corresponding to the same settlement of 25 mm. It is the measure of load sharing percentage of a pile within a group. The figure shows that central pile carries maximum load and load sharing percentage decreases with increase in distance of pile position from center of pile group where load is applied.



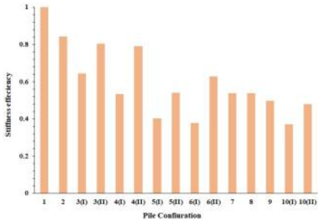


Fig. 10. Normalized pile load distribution for linear pile groups

Fig. 11. Stiffness efficiency of different pile groups

Though load-settlement response of pile groups (Fig. 8 and 9) clearly shows the interaction effect between piles in a group in terms of average load carrying capacity of pile in a group as compared to individual pile, the effect of interaction has greater influence on pile group stiffness. Relative stiffness of pile group can be expressed by the term “stiffness efficiency” which is defined as ratio of settlement of single pile to that of pile group under action of mean axial load. Fig. 11 compares group settlement ratio of all cases for a settlement of 25 mm or 2% of pile diameter. Stiffness efficiency is always found to be less than unity. With increasing number of piles in group, stiffness efficiency decreases exhibiting increase in interaction effect within piles. It is further observed that linear arrangements of pile groups show considerably lower stiffness efficiency.

3.2 Estimation of Group-efficiency

On basis of design codes, variety of theoretical techniques can be used to evaluate load carrying capacity of pile foundations. However, most trustworthy approach for estimating pile capacity is thought to be using load test data from piles that have already been built on site. Ultimate bearing capacity of pile may be conventionally defined as load at which the pile head settles at a certain value traditionally 10% of pile diameter (Salgado 2008 [6]). There are several other established methods available to estimate pile capacity which include Davisson offset limit method (Davisson 1972 [7]), Chin-Kondner extrapolation method (Chin 1970 [8]), Hansen 80%-criterion (Fellenius 2001 [9]), Decourt extrapolation method (Abdelrahman et al. 2003 [10]) etc. For estimation of group capacity from load settlement curve, equal settlement criteria were adopted in majority of previous research investigations (Dai et al. 2012 [11]; Nasrollahzadeh and Hataf 2019 [12] etc.). IS 2911: Part 4 (2010) [13] also recommends criteria to estimate safe load carrying capacity from load settlement curve of pile load test. As per the guidelines given in IS code, maximum settlement values to be considered to arrive final or safe load carrying capacity of single pile and pile group are 12 mm and 25 mm respectively. In this study, to calculate load capacity, 3 conditions of settlements are considered. For calculation of group efficiency, capacities corresponding to low settlement (25 mm ≈ 2% of diameter) for both single pile and pile groups are considered (Criteria A). Moreover, group efficiencies are also calculated with the criteria of 12 mm (≈ 1% of diameter) settlement for individual pile and 25 mm settlement for pile groups to obtain safe capacity (Criteria B). However, ultimate load capacity in this study is estimated based on well accepted 10% relative displacement criteria, and group efficiency has been calculated accordingly (Criteria C). Established formulas from some published literatures which are also used to estimate the group efficiency are as follows:

Converse-Labarre equation (Bolin 1941 [14]):

$$Cg = 1 - \frac{\tan_{-1}\left(\frac{d}{Sp}\right)}{90} \; X \; \left[\frac{(n_2 - 1)\, K\, n_1 + (n_1 - 1)\, K\, n_2}{n_1\, K n_2}\right] \tag{1}$$

Seiler and Keeney Method (1944) [15]:

$$Cg = \left\{ 1 - \left[\frac{11\, K\, Sp}{7\, K\, (Sp^2 - 1)}\right] X \left[\frac{n_1 + n_2 - 2}{n_1 + n_2 - 1}\right]\right\} + \left[\frac{0.3}{n_1 + n_2}\right] \tag{2}$$

Das (2015) [16]:

$$Cg = \frac{2\, K\, (n_1 + n_2 - 2)\, K\, Sp + 4d}{p\, K\, n_1\, K\, n_2} \tag{3}$$

Where p stands for the perimeter of the cross section of the pile group.

Los-Angeles Group action method (Das 2015 [16]):

$$Cg = 1 - \frac{d}{\pi\, K\, Sp\, K\, n_1\, K\, n_2} \; X \; [n_1 \; X \; (n_2 - 1) + n_2 \; X (n_1 - 1) + \sqrt{2} \; X \; (n_1 - 1) \; X \; (n_2 - 1)] \tag{4}$$

McCabe-Lehane Method (2006) [17]:

$$Cg = \frac{\left(\frac{\mathrm{Bg}}{\mathrm{Bp}}\right)^{0.66}}{\mathrm{n}} \tag{5}$$

Where Bg and Bp are the diameter of pile envelop of the pile group and the diameter of the pile respectively and N is the number of piles in group. Here, Cg was termed as stiffness efficiency.

Poulos and Davies (1980) [18]:

$$\frac{1}{\mathrm{Cg}^2} = 1 + \frac{(\mathrm{N})^2 \mathrm{Q_0}^2}{\mathrm{Q_B}^2} \tag{6}$$

Where N is number of piles in group. Q_0 & Q_B are ultimate capacity of single pile and the ultimate capacity of block of piles in group respectively.

Sayeed and Baker (1992) [19]:

$$Cg = 1 - (1 - \eta_s{}' k)\rho \tag{7}$$

Where, $\eta'_s = 2X \frac{[(\mathrm{n_2}-1)\mathrm{Sp}+\mathrm{d}]+[(\mathrm{n_1}-1)\mathrm{Sp}+\mathrm{d}]}{\pi \; \mathrm{K} \; \mathrm{d} \; \mathrm{K} \; \mathrm{n_1} \; \mathrm{K} \; \mathrm{n_2}}$, k = group interaction factor = 1 (assumed in the calculation) and ρ = friction factor = Q_s/Q_o; where, Q_s & Q_o are friction capacity and total static capacity respectively of single pile.

In general, n_1 and n_2 denotes the number of rows and number of piles per row respectively, d is diameter of pile and Sp is spacing between two consecutive piles.

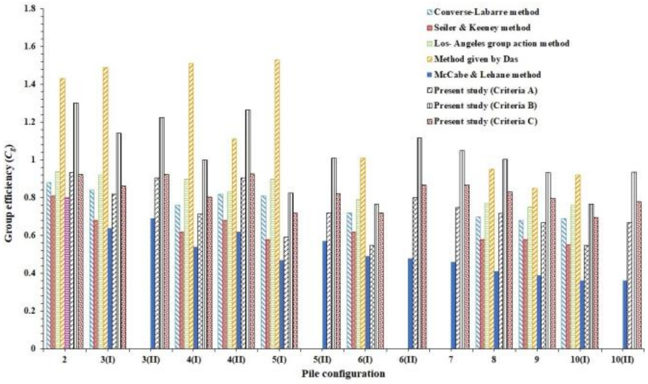


Fig. 12. Comparison of group efficiency (C_g) values obtained from the present study and different formulas with respect to geometry available in the literature

Fig. 12 and 13 shows comparison of efficiencies of pile groups calculated from above mentioned formulas and from results of the present study. Fig. 12 considers formulas related mainly to geometric arrangement of piles in group. whereas Fig. 13 considers formulas where the strength properties of different soil layers are involved. With respect to the geometric configuration of pile groups, it is witnessed that efficiencies calculated for given pile groups by criteria A of present study are closely conforming to efficiency values calculated using Seiler & Keeney method and Los-Angeles group action method. Furthermore, results obtained according to Criteria B conforms to those calculated using method provided by Das in majority of cases.

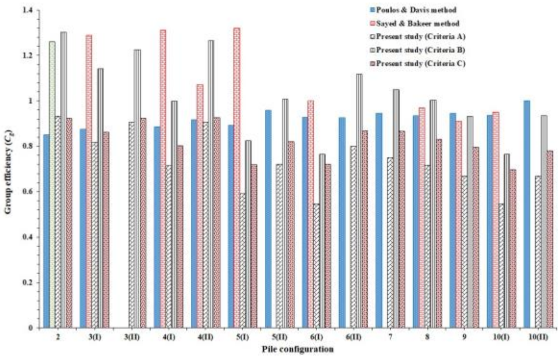


Fig. 13. Comparison of group efficiency (C_g) values obtained from the present study and different formulas with respect to geometry and soil properties available in the literature

For the established formulas requiring input of soil parameters as well the geometric configuration, group efficiencies calculated as per criteria A of the present study show a close relation with values calculated using method proposed by Poulos & Davies whereas, results as per criteria B agrees with values determined using method given by Sayed & Bakeer.

4 Constitutive Relationship

To understand the variation in response of soil with respect to Constitutive Relationship in this numerical study, soil behavior is represented either by Mohr-Coulomb model (MCM) or Hardening Soil model (HSM), whereas behavior of the pile and pile cap are linearly elastic. MCM results elastic perfectly-plastic relation, HSM follows hyperbolic stress-strain relation between axial strain and deviator stress in standard drained triaxial test. To define stress-strain relationship, MCM requires at least two out of the four elastic parameters i.e., Young's modulus (E), Poisson's ratio (v), oedometer stiffness (E_{oed}) and Shear modulus (G) whereas HSM requires three stiffness parameters i.e., triaxial primary loading stiffness at half of the maximum deviator stress (E_{50}), the triaxial unloading-reloading stiffness (E_{ur}) and the oedometer loading stiffness (E_{oed}). E_{50} and E_{ur} are the stress dependent stiffness moduli whose relationship with confining stress, p is controlled by parameter m. In Plaxis, stiffness values (E^{reff}) for HS soil model are to be provided from results obtained from triaxial test at an appropriate reference confining stress (p^{ref}). By utilizing the 'Soil Test' facility available in Plaxis, soil parameters utilized for HSM were reasonably approximated based on correlation of drain triaxial compression test results obtained for MC and HS soil models. Obtained correlations for all considered soil strata are presented in Fig. 14. Fig. 15 compares load-settlement curve of single pile and three pile groups (2, 8 & 9 piles) using both MCM and HSM for analysis. From curves shown in Fig 15, it is revealed that HSM predicts slightly higher load carrying capacity as compared to MCM at a certain value of settlement for all the cases. However, for low settlement values (less than 7 mm), difference in load between these two models were found to be insignificant.

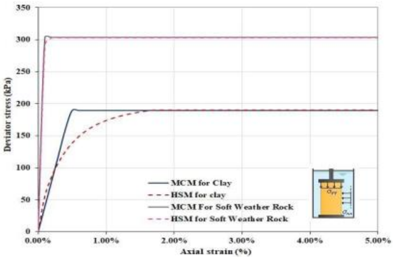


Fig. 14. Correlation of MC and HS Soil model for Stress-strain relationship

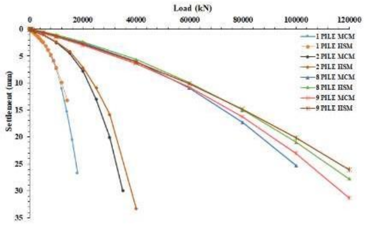


Fig. 15. Comparison of settlement response against vertical load with respect to constitutive relationship of soil

5 Conclusion

The interaction effects and load-carrying behavior of groups of pile subjected to concentric vertically applied load, are studied numerically using Plaxis. For this purpose, various configurations of pile groups were considered. Based on results obtained from numerical analysis, following conclusions are drawn with respect to the soil parameters considered in study:

- The layout of piles affects group efficiency of pile groups with same number of piles. Load bearing capability of pile groups with compacted arrangements is higher than that of pile groups with linear arrangements. Moreover, in group of piles with single row layout, pile that is farthest from the centre shares least load and vice versa.
- Mesh size variations do not significantly affect output results.
- For particular soil profile considered in this study, group efficiency value is always less than unity for equal settlement criteria (within the range of safe load value) no matter the number of piles in group where pile spacing remains constant (2.5D). Furthermore, efficiency value shows a reversal in trend when compared to total number of piles in group.
- Group efficiency calculated from the safe load capacity obtained from the criteria suggested in IS 2911 (Part-4) are always close to unity for the pile group containing more than four piles and greater than unity for pile group having small number of piles (up to 4).
- Group efficiency values in terms of ultimate load capacity criteria lies between 0.7 to 0.95 irrespective of the number of piles in the group for pile spacing equal to 2.5D.
- Stiffness efficiency values are always less than unity and is function of pile configuration in the group. Pile interaction has much greater effect on stiffness efficiency than group efficiency.

- In comparison with two different constitutive relationship of soil models, namely Mohr-Coulomb model (MCM) and Hardening Soil model (HSM), HSM predicts marginally greater load carrying capacity at a higher settlement.

References

1. Barden, L., Monckton, M. F.: Tests on model pile groups in soft and stiff clay. Géotechnique 20(1), 94-96 (1970).
2. Briaud, J. L., Tucker, L. M., Ng, E.: Axially loaded 5 pile group and single pile in sand, In: Proceedings of 12th International Conference on Soil Mechanics and Foundation Engineering, Rio De Janeiro, 15(5), 1121-1124, ISSMGE report (1989).
3. Comodromos, E. M.: Response evaluation of axially loaded fixed-head pile groups using 3D nonlinear analysis. Soils and Foundations 44(2), 31–39 (2004).
4. Naveen, B. P., Sitharam, T. G., Vishruth S.: Numerical simulation of vertically loaded piles. In: Proceedings of Indian Geotechnical Conference (IGC 2011), (Paper No. N-118), 835-838, Kochi (2011).
5. Dey, A.: Calibration of a PLAXIS Finite Element Dynamic Model: Effect of Domain Width and Meshing Schemes/AES. In: Proceedings of 3rd Indian Young Geotechnical Engineers Conference (3IYGEC), pp. 127-132, Indian Geotechnical Society, New Delhi (2011).
6. Salgado, R.: The engineering of foundations. McGraw-Hill Education. 1st edn. (2008).
7. Davisson, M. T.: High-Capacity Piles. In: Proceedings of Soil Mechanics lecture series on Innovations in Foundation Construction, pp. 81-112. ASCE, Illinois section, Chicago (1972).
8. Chin, F. K.: Estimation of ultimate load of piles not carried to failure. In: Proceedings of 2nd Southeast Asia Conference on Soil Engineering, pp. 81-92, Singapore (1970).
9. Fellenius, B. H.: From strain measurements to load in an instrumented pile. Geotechnical News Magazine 19(1), 35 – 38 (2001).
10. Abdelrahman, G. E., Shaarawi, E. M., Abouzaid, K. S.: Interpretation of Axial Pile Load Test Results for Continuous Flight Auger Piles. In: Proceedings of the 9th Arab Structural Engineering Conference, pp. 791-801, Abu Dhabi, UAE (2003).
11. Dai, G., Salgado, R., Gong, W., Zhang, Y.: Load tests on full-scale bored pile groups. Canadian Geotechnical Journal 49, 1293-1308 (2012).
12. Nasrollahzadeh, E., Hataf, N.: Experimental and numerical study on the bearing capacity of single and groups of tapered and cylindrical piles in sand. International Journal of Geotechnical Engineering 16(4), 426-437 (2019).
13. IS 2911-Part 4: Code of Practice for design and construction of pile foundations – Load test on piles. Bureau of Indian Standards, New Delhi (2010).
14. Bolin, H. W.: The Pile Efficiency Formula of the Uniform Building Code. Building Standards Monthly 10 (1), 4–5 (1941).
15. Seiler, J. F., Keeney, W. D.: The Efficiency of Piles in Groups. Wood Preserving News 22 (11), 109–118 (1944).
16. Das, B. M.: Principles of Foundation Engineering. Cengage learning. 18th edn. Boston, USA (2016).
17. McCabe, B. A., Lehane, B. M.: Behavior of axially loaded pile groups driven in clayey silt. Journal of Geotechnical and Geoenvironmental Engineering 132, 401-410 (2006).
18. Poulos, H. G., Davis, E. H.: Pile foundation analysis and design. John Wiley, New York (1980).
19. Sayed, S. M. and Bakeer, R. M.: Efficiency formula for pile groups. Journal of Geotechnical Engineering 118(2), 278-299 (1992).