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Comparison of Limiting Equilibrium and Finite Element Analysis for Embedded Retaining Walls

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Abstract. A classic part of geotechnical engineering is the design of embedded retaining walls. They comprise of sheet pile walls, secant pile walls, diaphragm walls and similar and are important part of many civil structures. Embedded retaining walls commonly comprise steel sheet piling or concrete walls, built as diaphragm walls in slurry trenches or using piling methods. The performance and design of embedded walls has been debated extensively by many authors, whilst codes of practice aim to specify design procedures. This paper compares the results of a limit equilibrium approach with the results of finite element method. The finite element analysis was done in PLAXIS code and soil was modeled as Mohr-Coulomb. The soil was modeled as drained type with zero pore water pressures. The analysis of cantilever wall, single propped wall (CMRL-Case study) and multi-prop wall as given in CIRIA SP 95 for sheet pile cofferdams is done and the results were compared. The compared results show that both the limit equilibrium method used for analysis and the finite element approach gives similar prediction of the maximum bending moment. It also shows that the earth pressures in the passive state are matching at the top, i.e., just below the excavated level, but the earth pressures from the Plaxis decreases as it moves towards the toe.

Keywords: Retaining walls, diaphragm walls, earth pressure, finite element, soil-structure interaction.

1 Introduction

Embedded retaining walls comprise of sheet pile walls, secant pile walls, diaphragm walls, etc., and for many civil structures are the most important part. The design of embedded retaining walls is a part of classic geotechnical engineering and 'limit equilibrium approach' is the most common approach for their design. Limiting earth pressures is the typical assumption of this approach and the wall should satisfy horizontal and moment equilibrium as well. The calculation of the embedment depth D and the prop force P for a typical propped cantilever wall shown in Fig. 1 uses two equations from two equilibrium equations. Subsequently, the bending moment (BM) distribution in the wall can be determined and the maximum value is taken as the design moment M. Earth pressure (EP) distribution acting on the wall is the critical matter in any method of limit equilibrium. Although the distribution depends on the mode of failure and for different modes of failure it may vary significantly, the linear variation of earth pressure with depth is assumed in most limit equilibrium applications. Earth pressure coefficients from classic earth pressure problem gives the value of limiting active and passive EP's acting behind and in front of the wall. This variant of limit equilibrium is often referred to as the 'free earth support' method [1]. Nevertheless, several other EP distributions are possible to consider. A method provided by Brinch Hansen [2] is remarkable in this regard. Although the method has the same basic components, it is not regarded as limit equilibrium approach. This method allowing consideration of a larger range of EP distributions each corresponding to a mode of failure is the main difference from limit equilibrium approach. Though relatively complicated in terms of calculations, this method is not broadly used outside of Brinch Hansen's native Denmark. Simpson and Powrie [3] provides the additional details on historical developments for the design of embedded retaining walls using limit equilibrium and other approaches.

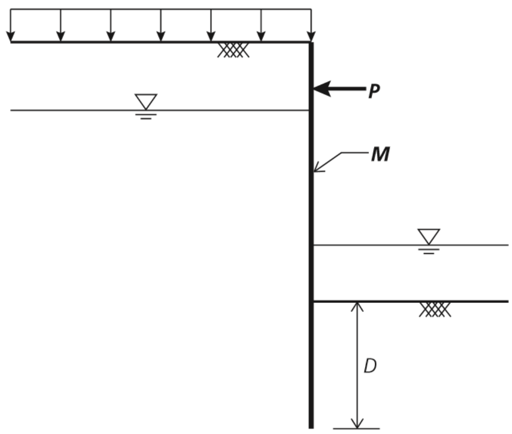

Fig. 1. A typical propped retaining wall.

By means of computerized implementations, classic limit equilibrium approach continues to be used extensively. At the same time as finite-element method (FEM) provides several advantages, tendency to use FEM is increasing. However, the practical application of FEM is not without complications and therefore is regarded as analysis tool rather than a design tool. As such, an obvious approach to the task of determining, say, the three design parameters M, P and D for a wall such as the one shown in Fig. 1 does not suggest itself readily. Furthermore, the way partial coefficients should be applied has been, and remains, the subject of some debate [4, 5, 6]. The most critical issue in the design is the determination of embedment depth by means of staged excavation analysis [7]. Once the minimum allowable embedment depth is determined, the maximum moment and prop force are used as a basis for the selection of the wall profile and design of the horizontal support system.

Limit equilibrium method was first introduced by Coulomb to calculate the force of a fill on a retaining wall. Later the theory was extended to infinite body by Rankine and earth pressure theories were developed. Subsequent developments were made by by Fellenius, Terzaghi and others that results in making the limit equilibrium method a well utilized tool for stability calculations by practicing engineers. An arbitrary mechanism of collapse can be constructed with limit equilibrium method. Each element of mechanism should be in equilibrium, the whole mechanism is in equilibrium is the main assumption of the limit equilibrium approach. While in the implementation of Observational Method, finite element analysis has played a crucial role. An analytical tool is provided by FEM for making initial predictions and helps engineers to set "trigger" levels for ground movement during construction. FE model can be calibrated by using the observed behavior during early stages and can be revised as necessary. If alternative construction strategies need to be examined, this can be done in a suitable manner.

This paper presents sample calculations by limit equilibrium method and comparison of the results with that of plaxis. Cantilever wall is analyzed limit equilibrium and compared with plaxis results. Similarly, limit equilibrium analysis of single propped wall (CMRL-Case study) is carried out and compared with plaxis results. Limit equilibrium analysis of multi-prop wall as given in CIRIA SP 95 for sheet pile cofferdams is compared with plaxis results.

2 Cantilever wall analysis

Cantilever wall analysis was carried out by means of limit equilibrium method. Problem setup for excavation of 3 m is shown in Fig. 2. The figure illustrates the cross-section of the embedded retaining wall with soil properties and the corresponding typical lateral earth pressure diagram. A homogeneous cohesionless soil layer of φ value 33° and γ value 20 kN/m^3 is considered for the analysis. The water table is at the ground level (GL). As per the analysis, an embedment of 6.1 m was required for the given soil parameters. The maximum BM was found at the level of zero shear at Limiting Equilibrium Condition (LEC). Consequently, the finite element analysis was carried out in plaxis with input parameters given in Table 1.

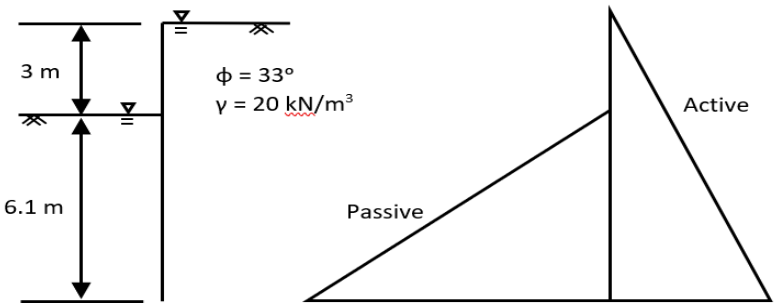

Fig. 2. Cross-section of embedded retaining wall with lateral earth pressure diagram.

Table 1. Plaxis input parameters for cantilever wall analysis.

Parameters	Units	Properties
Soil		
Identification		Sand and gravel
Saturated unit weight (γ_{sat})	kN/m^3	20
Young's modulus (E_{ref})	kN/m^2	3.5*10^4
Poisson's ratio (v)		0.35
Cohesion / angel of internal friction (c/φ)	kN/m^2/ deg	0 / 33
Plate		
Identification		D-Wall
Normal stiffness (EA)	kN/m	5.5*10^6
Flexural stiffness (EI)	kNm^2/m	8.4*10^4
Equivalent thickness (d)	m	0.42
Weight (w)	kN/m/m	12.5
Poisson's ratio (v)	-	0.15

The Fig. 3 compares the (a) lateral earth pressure acting on the wall in active and passive conditions, and (b) bending moment (BM) obtained from plaxis and limit equilibrium calculation.

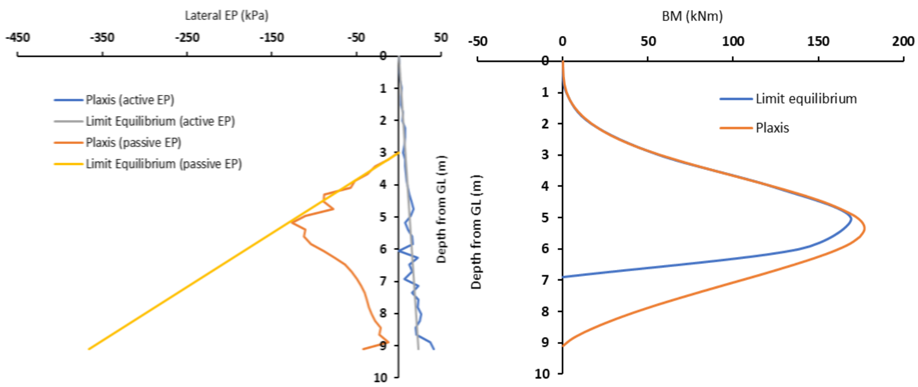

Fig. 3. Comparison between limit equilibrium and plaxis results: a) Earth pressure diagram, and b) Bending Moment diagram.

It can be observed that the active pressure as per plaxis is almost matching with the limit equilibrium pressure, which varies linearly with depth. The small variations towards the bottom can be because of the soil structure interaction and wall deformations. But in the passive side the plaxis and limit equilibrium values match in the beginning and the plaxis values reduces later. The limit equilibrium values are at the ultimate condition, which requires larger wall movements towards the toe. It doesn't happen in the working conditions. So only a part of the passive resistance is mobilized. In case of BM, the values almost match. The limit equilibrium calculation gives a value of 169 kNm and plaxis gives a value of 176 kNm. The reduction in passive resistance in plaxis probably causes the small increase in bending moment.

3 Analysis of propped walls

The analysis of single and multiple propped walls is discussed in this section. Single propped wall analysis is discussed considering the CMRL (Chennai Metro Railway) case history and multiple propped case is discussed considering the solved example from CIRIA SP 95 (Construction Industry Research and Information Association). The limit equilibrium analysis considering stage-by-stage method is compared with plaxis analysis.

3.1 Single propped wall (CMRL case study)

The multi-layered soil profile as per CMRL case history was used in the single propped wall analysis (see Fig. 4). The water table is at a depth of 4.71 m from the GL. For the excavation of 7.4 m an embedment of 4.4 m was provided for the retaining wall. The earth pressure diagrams were drawn, and the prop force, maximum BM and overall FoS was determined by limit equilibrium method. The depth of embedment was determined at LEC, and the prop force was calculated taking the sum of horizontal forces equal to zero at LEC. Then the level of zero shear and the maximum BM was determined. The overall FoS for the given embedment was also determined based on gross pressure method. Consequently, the finite element analysis was carried out in plaxis with input parameters shown in Table 2.

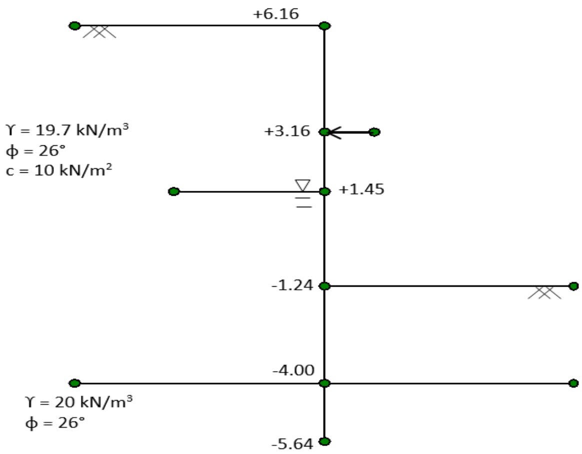

Fig. 4. Soil profile with level of excavation and prop.

Table 2. Plaxis input parameters for single propped wall analysis.

Parameters	Units	Value
Soil		
Identification		Sand and gravel
Dry unit weight(γ_{unsat})	kN/m^3	19.7
Saturated unit weight (γ_{sat})	kN/m^3	20
Young's modulus (E_{ref})	kN/m^2	2.29*10^4
Poisson's ratio (ν)		0.3
Cohesion / angel of internal friction (c/φ)	kN/m^2/ deg	0 / 35
Plate		
Identification		D-Wall
Normal stiffness (EA)	kN/m	2.37*10^7
Flexural stiffness (EI)	kNm^2/m	1.26E+06
Equivalent thickness (d)	m	0.80
Weight (w)	kN/m/m	2.00-10^1
Poisson's ratio (ν)	-	0.15
Struts		
Identification		Node to node Anchor
Normall stiffness (EA)	kN/m	2.38*10^5
Spacing (l_s)	m	1.5

A comparison of the limit equilibrium and plaxis pressure diagrams in active and passive condition is shown in Fig. 5. In case of plaxis, an increase in active pressure at the top and a decrease in the pressure below the prop is observed. Finally, towards the toe the limit equilibrium and plaxis earth pressures almost match. In the passive state, the earth pressures are matching at the top, i.e., just below the excavated level. But the plaxis earth pressures decreases as it moves towards the toe.

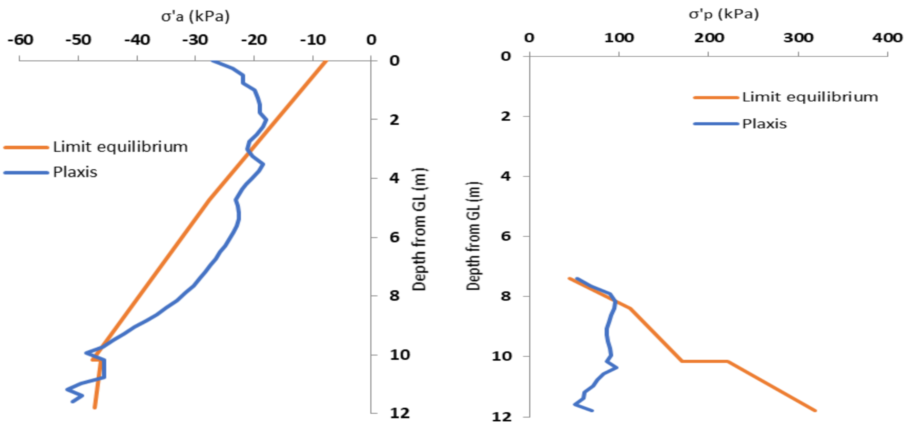

Fig. 5. Comparison of active and passive EP from limit equilibrium and plaxis results.

The difference in EP distribution from the limit equilibrium method are explained by various researchers. The following EP diagrams (Fig. 6) clearly illustrates the real and limit equilibrium pressure distribution. The results obtained from the case study is similar to them, plaxis is almost showing the real pressure distribution.

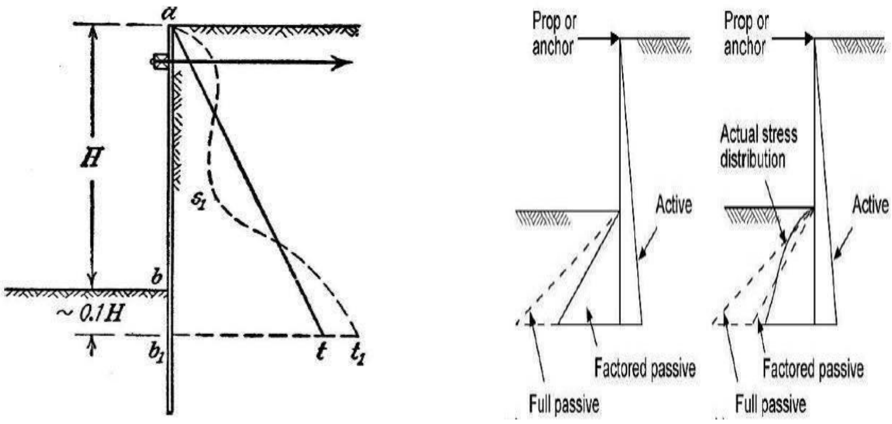

Fig. 6. Limit equilibrium and actual earth pressure diagrams (a) active (Terzaghi) (b) passive for rigid and flexible walls (Rowe)

In active case, the increase in the pressure at top is because of the inward movement of the wall due to prop. Due to the outward deformation of wall below the prop, the rigid prop results in arching of the soil. The arching of the soil increases the pressure on the prop and hence increases the prop load, as can be seen from the results. In the passive state (like in the cantilever case) large movement is required to develop the complete passive resistance. But the actual mobilised passive resistance will only be a part of the ultimate passive resistance. The arching of the soil and hence the decrease in active pressure reduces the bending moment of the wall. The reduction of passive resistance also reduces the overall FoS, compared to limit equilibrium result. Table 3 tabulates the results from limit equilibrium and plaxis analysis.

Table 3. Comparison of results from limit equilibrium and plaxis analysis.

	LE analysis	Plaxis analysis
FOS	1.88	1.72
BM	125 kNm	95 kNm
Strut Force	1287 kN	1395 kN

3.2 Multiple propped wall

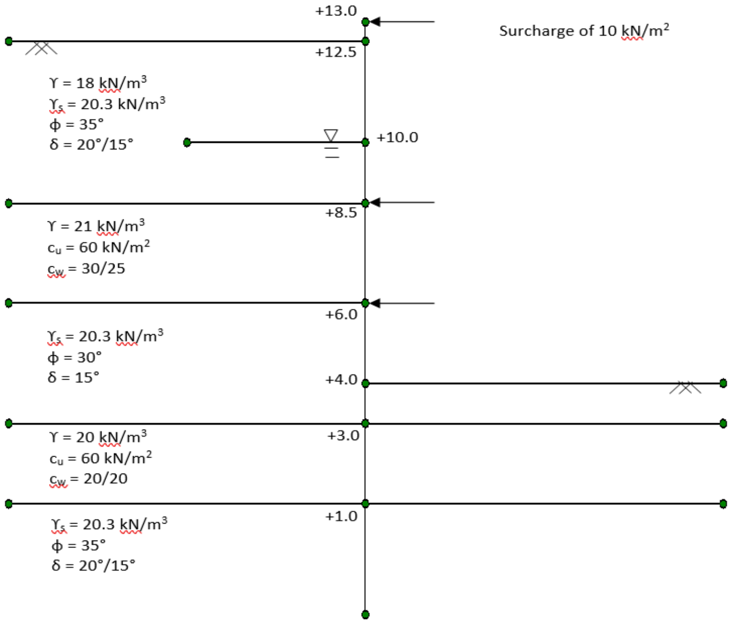

Fig. 7. Soil profile with levels of props and excavation.

The multiple propped wall analysis in CIRIA SP 95 [8] is compared with plaxis analysis. The limit equilibrium analysis is carried out using stage-by-stage method, considering the construction sequence and analyzing the wall at each stage. For the soil profile provided (see Fig. 7), active and passive EP's were calculated. The wall was analyzed for the first stage of excavation with single prop, considering the level of zero net pressure as the point of contraflexure. The prop load and the maximum BM at the level of zero shear was determined at this stage. For stage 2, the wall between the prop 1 and prop 2 was considered. A hinge is assumed at the level of prop 2 and the wall was analyzed for the load on prop 1 and the reaction on prop 2. Now for the wall below prop 2, the analysis was carried out as a single propped wall assuming a hinge at prop 2. The load on prop 2 and maximum BM at level of zero shear was determined at LEC. The adequacy of the embedment depth was also checked for the required FoS. Same procedure was repeated for stage 3 of the excavation. The above profile was modelled and analyzed in plaxis with input parameters given in Table 4.

Table 4. Plaxis input parameters for multiple propped wall analysis (CIRIA SP 95).

Parameters	Units	Value				
Soil		Layer 1	Layer 2	Layer 3	Layer 4	Layer 5
Depth (m)		13 to 8.5	8.5 to 6	6 to 3	3 to 1	1 to top
Identification		Sand and gravel	Firm clay	Silty sand	Soft/firm clay	Medium sand
Dry unit weight (γ_{unsat})	kN/m^3	18	18	18	18	18
Saturated unit weight (γ_{sat})	kN/m^3	20.3	21	20.3	20	20.3
Young's modulus (E_{ref})	kN/m^2	$4*10^4$	$5*10^4$	$5*10^4$	$5*10^4$	$5*10^4$
Poisson's ratio (ν)	-	0.3	0.3	0.3	0.3	0.3
Cohesion / angel of internal friction (c/φ)	kN/m^2/ deg	0 / 35	60	0 / 30	40	0 / 35
Plate						
Identification				D wall		
Normal stiffness (EA)	kN/m			3.79E+06		
Flexural stiffness (EI)	kNm^2/m			4.18E+04		
Equivalent thickness (d)	m			0.36		
Weight (w)	kN/m/m			7.00		
Poisson's ratio (ν)	-			0.25		
Struts						
Identification				Node to node anchor		
Normal stiffness (EA)	kN/m			6.37E+06		
Spacing (l_s)	m			9		

The results obtained from the two analysis are tabulated in Table 5. Due to the soil structure interaction and the wall deformations, redistribution of the pressure occurs. This results in a change in BM and prop loads. The horizontal stress distribution for rigid props on the retaining side of the wall is non-linear with load arching on to the relatively stiff prop [9]. Therefore, reduction in the bending moment occurs as lateral stress at the mid-section of the wall gets reduced. Because of the tendency of the upper part of the wall to rotate back into the retained ground, the rise in the lateral stress in the area of the prop may probably be noticeable in case if the wall is propped just below the [10].

Table 5. Limit equilibrium and plaxis results of the multiple propped wall (CIRIA SP 95).

Stage No	Prop Load 1 (kN)	Prop Load 2 (kN)	Prop Load 3 (kN)	Max BM (kNm)
Limit equilibrium results				
1	25			58.3
2	14.9	166.8		268.7
3	14.9	68.1	203.8	171.7
Plaxis results				
1	22.6			42

2	25.3	291.3		133
3	26.4	295.23	122.68	163

In the pile, reduction in bending moment occurs due to redistribution of pressure. In front of the wall the passive resistance close to the ground level may equal or surpass the limit equilibrium passive values and may decrease with depth towards the toe of the wall. The net result is a reduction in maximum bending moment compared with design based on a linear increase in limiting pressure with depth. This is however accompanied by an increase in the anchor loads [11]. (BS 8002)

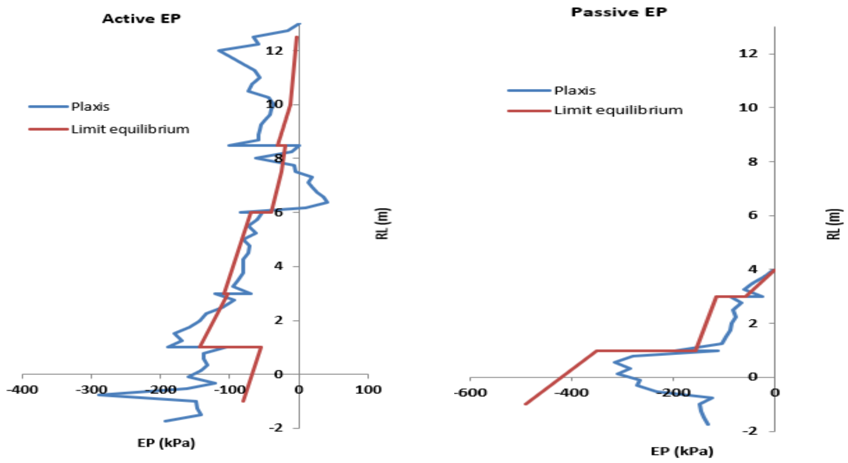

Fig. 8. Soil profile with levels of props and excavation.

The EP diagrams based on the limit equilibrium calculations and plaxis are presented in Fig. 8. These diagrams are like that of the single propped case, the plaxis results showing some variation from the limit equilibrium EP. As discussed in the single propped case, the variations here also can be due to the arching effect onto the rigid prop and the wall deformations.

4 Conclusions

The analysis of cantilever wall, single propped wall (CMRL-Case study) and multi-prop wall as given in CIRIA SP 95 for sheet pile cofferdams is done using limit equilibrium approach and the results obtained are compared with those from the plaxis analysis.

In case of cantilever wall, it was observed that the active pressure as per plaxis is almost matching with the limit equilibrium pressure with small variations towards the bottom. But in the passive side, the plaxis and limit equilibrium values match in the beginning and the plaxis values reduces later. As for BM, the values almost match.

In case of single propped wall, an increase in active pressure at the top and a decrease in the pressure below the prop is observed for plaxis result. Finally, towards the toe the limit equilibrium and plaxis earth pressures almost match. In the passive state, the earth pressures are matching at the top, i.e., just below the excavated level. But the plaxis earth pressures decreases as it moves towards the toe.

In case of multi-propped wall, the earth pressure diagrams are similar to that of single propped wall. The plaxis results shows some variation from limit equilibrium earth pressures which may be due to the arching effect onto the rigid prop and the wall deformations.

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