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The Importance of Principal Stress Rotation in Transportation Geotechnics and Associated Constitutive Models.

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Abstract. Among all the transportation modes worldwide, railways are pivotal in catering to the ever-growing population's increasing demands. The most recurring challenge in the field of railways is track failure. Out of all the track layers, the subgrade is the weakest layer. Therefore it becomes necessary to study the performance of railway track subgrade in detail. The laboratory experiments to study subgrade behaviour under moving loads are primarily conducted on cyclic triaxial equipment. However, the Principal Stress Rotation (PSR) phenomenon affects the actual field condition. PSR is the rotation of principal stresses due to cyclic vertical and shear stresses resulting from a moving load. The axisymmetric loading condition applied in the cyclic triaxial apparatus fails to capture the true stress path in the soil underneath railway tracks and pavements. Therefore, it is imperative to study the effect of PSR to understand soil deformation behaviour when subjected to moving traffic. Several constitutive models have also been developed to understand the soil deformation behaviour under stress paths with Principal Stress Rotation. The common approaches to incorporating PSR into classical models include using the Transformed stress method, employing additional strain mechanisms due to PSR, relating plastic modulus and dilatancy to inherent anisotropy, and incorporating fabric evolution law to consider the effect of induced anisotropy, to mention a few. The current paper critically reviews the importance of considering Principal Stress Rotation in Transportation Geotechnics, especially in the case of rail track subgrade. Additionally, it aims to provide an overview of some of the constitutive models studying the effect of PSR on sands and clays. Comparing and analysing different models can help understand the primary mechanism behind PSR and its impact on soil deformation.

Keywords: Principal Stress Rotation, rail track subgrade, moving loads, constitutive models.

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1 Introduction

Railways play a vital role in the commercial growth of any country. It acts as an economic lifeline of the country by hauling goods and transporting passengers over long distances. According to the Rail Transport Global Market report 2022, the global rail market is expected to have a compound annual growth rate of 7.4% by 2026. High-speed railways are considered a better alternative to short-distance air travel, and freight rails are always an energy-efficient solution to transport cargo across places. Therefore, it is imperative to have a robust rail transport network in terms of its form, safety, quality of travel, and time efficiency.

The railway substructure consisting of ballast, sub-ballast and subgrade is the most critical layer. Common problems associated with substructure include track buckling (Ngamkhanong et al. 2021), differential track settlement (Selig and Waters 1994), ballast degradation and fouling (Tennakoon et al. 2012), and subgrade instabilities like excessive plastic deformation, subgrade fluidisation and progressive shear failure (Li and Vanapalli 2021; Nguyen and Indraratna 2022; Sánchez et al. 2014). These issues lead to the substandard performance of rail networks resulting in delays, accidents and high maintenance costs. Most often, the rail track substructure, especially the subgrade, is overlooked compared to the superstructure (Selig and Waters 1994; Burrow, Bowness, and Ghataora 2007).

Laboratory studies that simulate traffic conditions on subgrade material, either actual field soils or artificially prepared soil specimens, were mainly conducted using triaxial equipment. Even the conventional track design methods often rely on the results of repeated load tests, static triaxial tests and cyclic triaxial tests. The British Railways method of predicting the granular layer thickness used triaxial compression test results to determine the threshold stress level of the subgrade (Heath et al. 1972). The Li-Selig design method used cyclic triaxial test results to assess cumulative plastic strain in the soil at different deviator stresses and proposed empiricial formulation to predict the granular thickness (Li and Selig 1996, 1998). The North American Railway method, assumed allowable subgrade pressure to be 138kPa neglecting different soil conditions, whereas the Canadian method considered Cassagrande soil classification to determine the allowable bearing capacity of subgrade soils while neglecting the effect of repeated loads on the track (Sayeed 2016). However, these design methods neglected the actual field stress paths.

In the field, when a train passes over a track or a vehicle moves over pavement, the subgrade soil experiences complex stress paths. Various studies found that a moving load approaching, receding or stopping cause the rotation of the principal stresses due to the shear stress reversal in soil mass. This phenomenon is called Principal Stress Rotation (PSR) (Brown 1996; Powrie, Yang, and Clayton 2007). Figure 1 shows the rotation of Principal stresses due to moving wheel load.

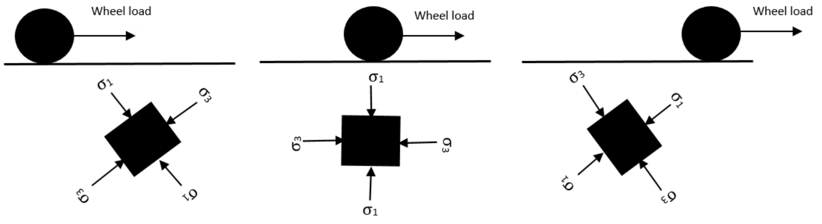

Fig. 1. Two dimensional representation of Principal Stress Rotation under a moving wheel load (modified from Wang et al. 2019)

The design and studies related to rail track engineering have often disregarded the presence of PSR and its associated effects. This paper attempts to provide a critical analysis and discussion of the existing studies relevant to Principal Stress Rotation under railway tracks. The importance of PSR must be discussed in much greater depth to benefit the scientific community working in this area including experimental studies and the constitutive models that incorporate the concept of PSR.

2 Experimental studies related to traffic loading

The majority of studies conducted on soil samples subjected to traffic loads were undertaken using cyclic triaxial equipment where traffic loading was replicated using a repeated vertical load (Shahu and Kameswararao 2000; Wang et al. 2013; Shahu, Yudhbir, and Rao 1999; Singh, Indraratna, and Nguyen 2021). Others used two-way cyclic loading to replicate the principal stress axis reversal (Yasuhara, Hirao, and Hyde 1992). However, this experiment instantaneously changes the principal stress direction from 0° to 90° and fails to replicate the field conditions. Others adopted variable confining pressure approach, i.e., cyclic confining and deviator stresses to mimic traffic-induced stresses (Cai et al. 2013; Gu et al. 2016), and it was pointed out that tests with cyclic confining pressure have a more significant influence on the deformation of soils. Some studies were conducted using the True triaxial equipment, where the intermediate principal stress (σ_2) can be cycled in addition to the major principal stress (σ_1) (Gu et al. 2018; Gu et al. 2019).

Figure 2 shows the stress paths in the deviator and torsional stress space for the case of wave and traffic loading with the amplitude of cyclic deviator stress (q_{cyc}) and the value of the angle the principal stress makes with the vertical axis (α). In the case of traffic conditions, the deviator stress is changed simultaneously with the direction of principal stress, unlike the wave loading, which consists of only pure principal stress rotation with constant deviator stress. Therefore, the stress path for traffic loading conditions is a cardioid shape in the $(\sigma_z\text{-}\sigma_\theta)/2\text{-}\tau_{z\theta}$ plane.

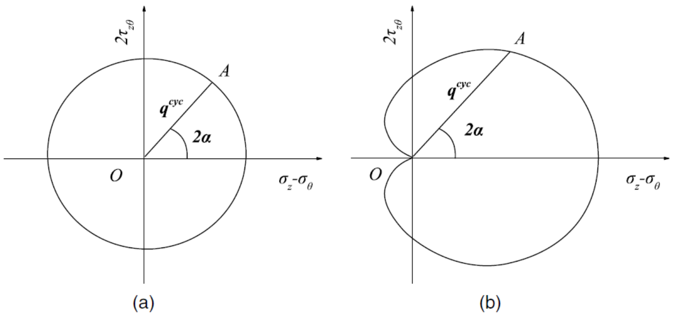

Fig. 2. Typical stress paths in the deviator and torsional stress space for (a) Wave loading and (b) traffic loading (Cai et al. 2018)

2.1 Influence of Principal Stress Rotation

The principal stress rotation with cyclic deviator stress significantly affects the permanent strain accumulation, pore pressure generation, and stiffness degradation in soil (Guo et al. 2016; Wu et al. 2017; Cai et al. 2018). Several studies were conducted on the sand and sand-clay mixtures to understand the effect of PSR (Fedakar, Cetin, and Rutherford 2021; Cai et al. 2015; Gräbe and Clayton 2009). The tested specimens subjected to PSR using Cyclic Hollow Cylindrical Apparatus developed higher axial strain than those subjected to cyclic triaxial samples (Fedakar, Cetin, and Rutherford 2021; Cai et al. 2017; Guo et al. 2018). The increase in deformation is attributed to the rotation of major principal stress axes. At the micromechanical level, the softening response is assumed as a result of the change in the principal direction of soil fabric which follows the rotation of the principal axis (Qian et al. 2016). The studies conducted on Wenzhou clay at different values of vertical stress ratio and shear stress ratio show that even the low vertical stresses can generate higher axial strains when subjected to higher shear stresses. It was also indicated that higher shear stresses can lead to accelerated permanent strain (Wu et al. 2017). There is an increase in axial strain with an increase in the shear stress ratio, and the effect is more prominent in the case under a higher vertical stress ratio (Cai et al. 2018). Figure 3 shows the comparison of vertical strains developed in Cyclic Hollow Cylinder (CHC) and Cyclic Triaxial tests (CT) for a sand-clay mixture and Wenzhou soft clay at Vertical Cyclic Stress Ratio (VCSR) of 0.15 and 0.143 respectively. It can be observed that vertical strains developed in CHC tests are significantly higher for the sand-clay mixture (with 20% clay) when compared to CT tests. The result of Wenzhou clay also proves that CHC tests produce higher strains compared to CT tests.

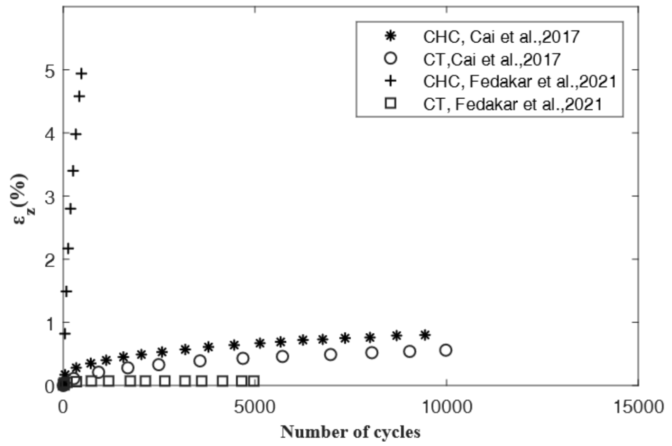

Fig.3. Comparison of vertical strain results from Cyclic Hollow Cylinder tests (for η=1/3) and Cyclic Triaxial tests (modified from (Fedakar, Cetin, and Rutherford 2021; Cai et al. 2017))

The undrained K0-consolidated Cyclic Triaxial (CT) and Cyclic Hollow Cylinder (CHC) experiments on intact clay samples showed that the pore pressure buildup in CT is less than that in CHC experiments, even when subjected to the same vertical stress ratio confirming the effect of PSR (Guo et al. 2018). A similar observation is found in another study (Qian et al. 2019) where undisturbed Shanghai Clay samples were subjected to cyclic loading with varying deviator stress (q), intermediate principal stress ratio (b) and PSR. It shows that PSR has a more significant effect on pore pressure generation than the intermediate stresses. In the case of sand samples also, higher deviation of principal stresses from the vertical (α) and b values generate higher pore pressure (Yoshimine, Ishihara, and Vargas 1998). A study on reconstituted materials consolidated to the same overconsolidation ratio (OCR) showed different pore pressure responses when tested under different loading conditions (no PSR and with PSR). For a mixture of sand, silt and clay, the pore pressure generated during tests with stress rotation at 200 cycles was approximately five times higher than those without PSR (Gräbe and Clayton 2009). Figure 4 shows a comparison of pore pressure ratios for Wenzhou and Shanghai clay obtained from CHC and CT tests. It can be seen that the PSR in the Cyclic Hollow Cylinder tests generates higher pore pressure.

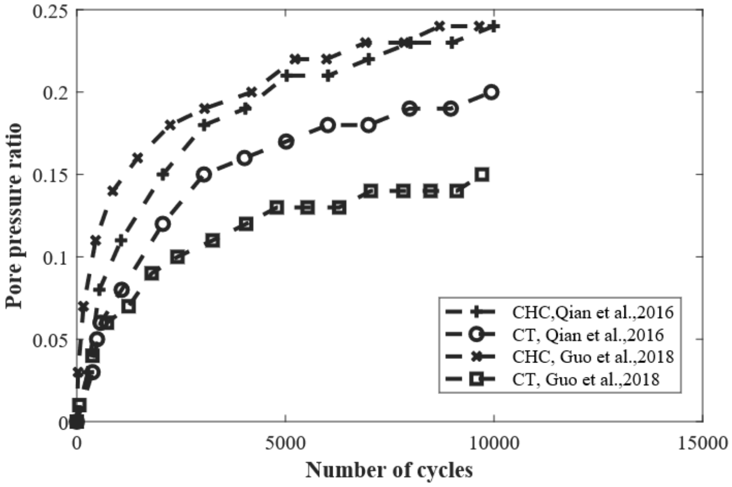

Fig.4. Comparison of pore pressure ratio from Cyclic Hollow Cylinder tests and Cyclic Triaxial tests (modified from Guo et al. 2018 and Qian et al. 2016)

Stiffness degradation can be observed during the Principal Stress Rotation. For undisturbed clay samples, the axial stress-strain hysteresis loops overlap and forms a single line for low CSR irrespective of the shear stress ratio (η). However, upon increasing CSR, the hysteresis loops widen at a higher number of cycles, and the effect of η becomes significant (Cai et al. 2018). It is also evident that the degradation of soil increases with η. Principal Stress Rotation also influences the deformation behaviour of sand-clay mixtures where PSR and clay content influenced the hysteresis loop area. The axial stress-strain hysteresis loops at different cycles demonstrates that at higher clay content, the specimens using CHC showed opened hysteresis loops which indicate irrecoverable strain development (Fedakar, Cetin, and Rutherford 2021). Figure 5 shows the axial stress-strain hysteresis loop for Wenzhou clay and sand-clay mixtures with 20% clay. It can be inferred from the plot that higher shear stress ratio increases the area of the hysteresis loops, irrespective of the soil types.

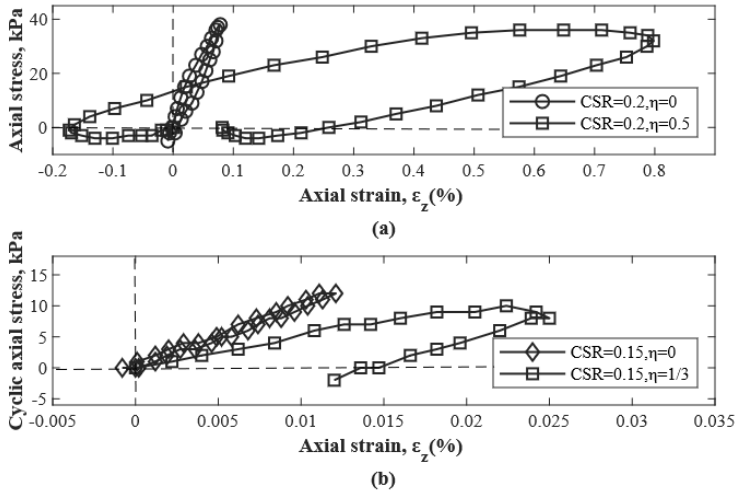

Fig.5. Axial stress-strain hysteresis loop for (a) Wenzhou clay (modified from Cai et al. 2018)

(b) sand-clay mixture with 20%clay (modified from Fedakar, Cetin, and Rutherford 2021**)**

For Wenzhou clay, it was found that the vertical resilient modulus (M_{rz}) decreases with the number of cycles and M_{rz} was lower for samples tested in the Hollow Cylinder apparatus due to the presence of Principal Stress Rotation. Based on vertical stress-strain hysteresis loops, samples subjected to higher shear stress ratios cause more significant energy dissipation under the same VCSR (Guo et al. 2018). For Shanghai soft clay, the stiffness gradually decreased towards instability under PSR in contrast to the stable behaviour observed in those without PSR (Qian et al. 2019).

For Hollow Cylinder Apparatus, there are two types of stress-strain loops (a) axial stress-strain loop and (b) torsional shear stress-strain loops (Qian et al. 2019; Qian et al. 2016). The secant moduli in the i^{th} cycle of the axial shear stress-strain and shear stress-strain curve were defined as axial shear stiffness, G_i and torsional shear stiffness, G^*_i respectively. With increasing the respective strain, the slope of the torsional shear stress-strain loop decreases rapidly compared to the slope of the axial stress-strain loop. This indicates that more cyclic degradation is evident in the torsional shear stress-strain loop, which results in an increased anisotropic behaviour of clay under repeated traffic loading. Some researchers indicate that non-coaxiality can be the reason for the anisotropic degradation phenomenon (Qian et al. 2016). Non- coaxiality is the non-coincidence of stress direction and strain increment direction under PSR due to the change in torsional shear stress rather than axial stress (Qian et al. 2019).

Figure 6 illustrates the phenomenon of non-coaxial behaviour in the soil under cyclic loading. The angles indicated in the figure represent the following:

α_{σ}: inclination angle of deposition direction with respect to the major principal stress direction

$\alpha_{d\sigma}$: inclination angle of major principal stress increment direction

α_{de}: inclination angle of major principal strain increment direction

β: angle between stress and strain increment

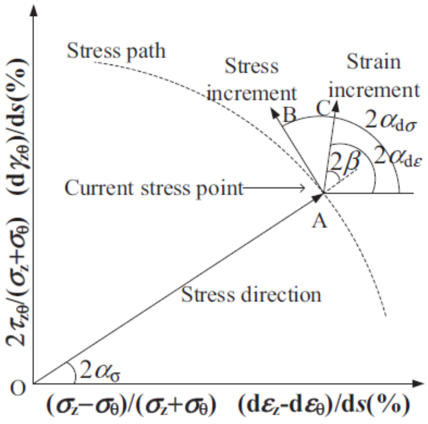

Fig.6. Non-coaxial behaviour (Qian et al. 2019)

If the direction of strain and stress increment coincide, then the principal direction of strain increment is determined solely from stress increment under elastic response. In the case of perfectly plastic material behaviour with isotropic hardening, the strain increment coincides with the stress direction. It was pointed out that strain increment direction depends on stress and stress increment direction when PSR is involved. The significant influence of PSR is that there is a non-coincidence of principal stress and principal plastic strain increment directions (Qian et al. 2019; Gutierrez, Wang, and Yoshimine 2009). Table 1 summarises the published experimental study conducted on Cyclic Hollow Cylinder equipment by various researchers.

Table 1. Summary of experiments studying Principal Stress Rotation under traffic loads

Author and Year	Soil studied	Test conditions	Remarks	Effect of PSR
Grabe and Clayton,2009	A reconstituted mixture of sand, silt and clay	Vertical total stress cycles between 30 and 60kPa; Shear stress cycles between -7 and 7 kPa	The effect of PSR on the permanent deformation of reconstituted materials is studied.	It was found that PSR significantly affects pore pressure generation and permanent deformation of the materials tested.
Xiao et al., 2014	Nanjing soft clay	Cyclic axial stress: 10.5kPa; Cyclic torsional shear stress: 0 and 1.25kPa; frequency: 0.29, 0.58 and 1.16Hz.	The effect of PSR on the pore pressure and permanent deformation of the normally consolidated medium plasticity soft clay is studied.	The pore pressure generation, cumulative deformation and axial stress-strain behaviour were seen to be influenced by PSR.
Qian et al.,2016	Shanghai Clay	Initial confining pressure,p_0: 100,150,200 and 250kPa; Dynamic stress ratio (q_{max}/p_0): 0.15-0.35; 0.15-0.4; 0.1-0.3; 0.07-0.25; Torsion shear stress ratio (τ_{max}/q_{max}): 0 and 0.25; Frequency:1 Hz	The shakedown approach was used to distinguish the undrained cyclic response of clay at different stress levels. Permanent strain and energy dissipation were used to study the plastic shakedown, cyclic plastic creep and ratcheting deformation patterns.	Tests with PSR seem to develop higher pore pressure and irreversible strain compared to triaxial tests.

Wu et al.,2017	Wenzhou soft clay	VCSR: 0.05, 0.15, 0.2,0.25,0.3,0.35,0.4; η ($\Delta\tau_{z\theta}^{ampl}/\Delta\sigma_{z}^{ampl}$): 0,0.25 and 0.5; frequency: 0.1Hz	The study highlighted the effect of initial shear stress on cyclic deformation of soft clay. Different η values were used to consider the influence of shear stress levels.	Shear stress levels increase the permanent strain and cyclic degradation with the situation worsened at higher VCSR. The allowable VCSR decreased with an increase in shear stress level.
Cai et al.,2017	Wenzhou soft clay	VCSR ranging from 0.006 to 0.274; η: 1/3 and 1/6	The study shows the influence of PSR on the vertical strain development in the soil.	The vertical strain developed in the Cyclic Hollow Cylinder apparatus is higher than that in the Cyclic triaxial test, thus highlighting the influence of PSR.
Cai et al., 2018	Wenzhou soft clay	CSR: 0.05-0.4; η: 0,0.25 and 0.5; frequeny:0.1Hz	Deformation of soil under different deviatoric stresses and shear stress values are conducted. The study develops an empirical model for degradation index, δ, which depends on CSR, η and the number of cycles.	It was observed that axial strain at a particular CSR is higher at high values of shear stress ratio. An increase in η increases the area of the stress-strain loop for a given CSR and the number of cycles.
Shi et al., 2018	Wenzhou soft clay	OCR: 1,1.5 and 2; Cyclic vertical stress (kPa) : 10,20 and 40; Cyclic shear stress (kPa): 2.5	The effect of OCR and PSR on the cyclic behaviour of marine clay is investigated.	The effect of OCR is such that the pore pressure and strain under stress paths with PSR are reduced with an increase in OCR.
Guo et al.,2018	Wenzhou soft clay	VCSR ranging from 0.006 to 0.274; η: 1/3 and 1/6	The study highlights the effect of PSR on the pore pressure generation, vertical strain, and resilient modulus of soil.	The tests with PSR showed significant vertical strain, pore pressure build-up and rapid degradation of vertical resilient modulus.
Wang et al., 2019	Wenzhou soft clay	VCSR:0.2; Cyclic shear stress ratio (η_t): 0,0.25; cyclic confining pressure ratio (η_d): 0, 0.25; frequency: 0.01,0.1 and 1Hz	The effect of Principal stress rotation, cyclic confining pressure and their combination on the long-term dynamic behaviour of soft clay was studied.	The degradation of resilient modulus was increased due to PSR due to an increase in accumulated strain.
Qian et al., 2019	Shanghai clay	Mean stress:150kPa; deviator stress: 4-30kPa and 8-60kPa; Intermediate principal stress parameter, b : 0,0.5,1; angular displacement rate: 0.2°/min.	The effect of PSR and intermediate principal stress parameter on the pore pressure generation, permanent deformation, stress-strain relation, shear stiffness behaviour and non-coaxiality were studied.	PSR showed influence on pore pressure generation and permanent deformation. Non-coaxiality, i.e. non-coincidence of principal directions of strain increment and stress, was seen due to PSR.
Thevakumar et al.,2021	Clay and sand mixture	CSR: 0.2 and 0.3; frequency: 0.1,0.5 and 1	The influence of PSR on the subgrade soil of rail tracks was studied.	The axial strains, pore pressure and stiffness degradation were higher in

				tests with PSR than in tests without PSR.
Pan et al.,2022	Wenzhou soft clay	VCSR: 0.05,0.15,0.25,0.3; η: 3.33,10,15.33,17.33; frequency:1Hz	The effect of PSR and different drainage conditions (partially drained and undrained) are studied.	In both undrained and partially drained conditions, PSR is seen to increase the permanent octahedral shear strain and permanent axial strain.

3 Constitutive models Capturing Principal Stress Rotation

This paper intends to give a brief description of the constitutive models which were developed to study the Principal Stress Rotation in soils. The common approaches to incorporating PSR into classical models include using the transformed stress method, employing additional strain mechanism due to PSR, relating plastic modulus and dilatancy to inherent anisotropy, and incorporating fabric evolution law to consider the effect of induced anisotropy, to mention a few. As explained in the previous sections, the additional volumetric and shear strain developed in the soil cannot be neglected to predict soil behaviors subjected to PSR.

A method by (Pande and Sharma 1983) to incorporate PSR into the constitutive model was to generalise the Multi Laminate model by activating/deactivating the sliding dilation/contraction on the planes. Explicit equations for yield functions were developed for each plane as a function of effective normal stress, shear stress and normal component of visco-plastic strain rate on that plane. These effective normal stress and shear stresses are a function of principal stress direction and magnitude. A constitutive model (Matsuoka and Sakakibara 1987) for sand and clay was developed considering the hyperbolic relation between stress ratio and shear strain in general coordinates. The model considered the shear strain as a function of ϕ_{mo} (mobilised frictional angle), α (angle between an arbitrary and principal stress plane and σ_m (confining pressure), which led to equations for shear strain increments due to shear, PSR and anisotropic consolidation in general coordinates, corresponding to the hardening function. The relation between stress ratio and strain increment ratio corresponds to the flow rule controlling the direction of strain increments.

A Bounding surface hypoplastic model was developed for sands which involved the dependence of plastic strain rate direction on the stress rate direction yielding an incremental non-linearity on stress-strain relation, simulating rotational shear stress. The model, however, involves a large number of parameters which renders the model's applicability difficult (Wang, Dafalias, and Shen 1990). In another study, the rotational loading mechanism was introduced to simulate the PSR. The stress ratio increment orthogonal to the stress ratio is simulated to produce additional plastic strain (Li and Dafalias 2004). The Bounding surface model introduced a non-coaxiality function involving loading interaction with soil fabric anisotropy. The function considers the effect of stress rate and principal stress direction on plastic strain rate direction (Lashkari and Latifi 2008).

To model, the effect of Principal stress rotation on the simple shear deformation of sand, anisotropy and non-coaxility were included in the model (Gutierrez, Wang, and Yoshimine 2009). To accommodate the impact of PSR, an anisotropic failure criterion, cross-anisotropic elasticity, plastic flow rule and a stress-dilatancy relation were adopted to accommodate the effect of non-coaxility (Gutierrez, Wang, and Yoshimine 2009). Another model by (Gao and Zhao 2017) based on anisotropic critical state theory considered fabric anisotropy and fabric evolution of sands. The fabric evolution law under PSR was simulated from the discrete element simulations. Under continuous principal stress rotation, the fabric of sand also rotates towards the loading direction. The model's plastic modulus and dilatancy relation were related to the fabric and fabric evolution. Material anisotropy is used to model PSR in an anisotropic unified hardening model within the framework of the elastoplastic theory. The inherent anisotropy is modelled using an anisotropic transformed stress method, and induced anisotropy is introduced utilising a fabric evolution law (Tian and Yao 2018). Table 2 gives a summary of different constitutive models considering Principal Stress Rotation effects.

Table 2. Constitutive models and Principal Stress Rotation

Author and Year	Model/Framework	Incorporation of PSR
Pande and Sharma,1983	Multi Laminate Model	Made by activating/deactivating the sliding dilation/contraction on various planes. .
Matsuoka and Sakakibara,1987	Hyperbolic stress-strain relation	Shear strain is considered a function of ϕ_{mo} (mobilised frictional angle), α (angle between an arbitrary and principal stress plane and σ_m (confining pressure).
Wang et al.,1990	Bounding Surface Hypo plasticity model for sand	The dependence of plastic strain rate direction on the stress rate direction leads to an incremental non-linearity for stress-strain relations.
Li and Dafalias, 2004	Bounding Surface model	By introducing a rotational loading mechanism to simulate the rotation of the principal stress direction.
Lashkari and Latifi,2007	Bounding Surface model	By introducing a non-Coaxility function to consider soil fabric anisotropy.
Gutierrez and Yoshimine	Hyperbolic stress-strain model	Anisotropy and Non-coaxility are incorporated into the model to study the effects of Principal stress rotation on the sand.
Gao and Zhao, 2017	Anisotropic critical state theory framework	A constitutive model was developed, which emphasised the role of fabric and fabric evolution to simulate the effect of PSR.
Tian and Yao, 2018	Anisotropic unified Hardening model	Inherent and induced anisotropy are considered to simulate the effect of stress rotation. Anisotropic transformed stress method and fabric evolution law are used in the model.

4 Conclusion

Principal Stress rotation and its effects on the dynamic soil behaviour were discussed. Based on the existing literature, the role of PSR on subgrade under railway track structures was often ignored. The influence of stress rotation on strain accumulation, pore pressure generation, and stiffness degradation needs to be further investigated to capture the actual deformation of the soil subjected to traffic loading conditions. In terms of the soil behavior under loading with and without PSR, this study found that rotation of stress direction generates higher axial strain, pore pressure and more degradation of soil confirming the importance of PSR.

Along with the experimental studies, constitutive models were developed to include the effect of stress rotation. It was made possible by generalising the Multi Laminate model, by developing equations for additional shear strain increment due to PSR, adopting hypo plasticity into the model, introducing rotational loading mechanism or introducing anisotropic and non-coaxiality functions into the model.

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