Visakhapatnam Chapter

Proceedings of Indian Geotechnical Conference 2020
December 17-19, 2020, Andhra University, Visakhapatnam

Probabilistic Analysis of Reinforced Soil Retaining Structures using FORM and Surrogate based Monte-Carlo Simulation

Ekansh Agarwal[1, 2], Anindya Pain[2] and Shantanu Sarkar[2]

[1] Research Scholar, Academy of Scientific and Innovative Research (AcSIR), Ghaziabad – 201002
[2] Geotechnical Engineering. Division, CSIR-Central Building Research Institute, Roorkee (India) -247667

Abstract. In the present study, reliability analysis of reinforced soil retaining structure (RSRS) under seismic conditions is performed. A reliability-based analysis of (RSRS) is performed in the present study using the random variable approach. The pseudo-static approach under the framework of limit equilibrium method of horizontal slices is employed to assess the internal stability of RSRS. The basic approach involves satisfying the Coulomb's failure criterion along the assumed failure surface. First Order Reliability Method (FORM) is employed to determine the reliability indices associated with various modes of failure. The system reliability index is also calculated using the same approach, considering the modes of failure to be connected in series. Tension mode is found to be the most critical mode of failure. A unique surrogate assisted Monte-Carlo Simulation (MCS) is carried out to validate the present formulation using Multivariate Adaptive Regression Splines (MARS) method. It is found that assumption of FORM may lead to the underestimation of the value of probability of failure of RSRS. Moreover, the surrogate based MCS outperforms the FORM in terms of computational efficiency and accuracy.

Keywords: Internal stability; FORM; Reliability; pseudo-static, MARS

1 Introduction

Reinforced soil retaining structures (RSRS) are widely used now a days. RSRS have all the attributes to eliminate the drawbacks of their conventional counterparts and are economical, easy to construct and offer a wide range of advantages to the designers and the architects. RSRS have become a crucial component of any slope stabilization project. These structures are durable and hence enhance the stability of the structure in which they are used. RSRS are analyzed using various analytical as well as numerical approaches for both, internal and external stabilities. The initial works on the design and analysis of RSRS were made by Leshchinsky et al. [1], Ling et al. [2] and Ling and Leshchinsky [3] using log-spiral failure surface in a limit equilibrium (LE) framework. Another approach of designing RSRS is the Horizontal Slice Method (HSM) which includes splitting the reinforced portion into a number of horizontal slices and then employing suitable equations of equilibrium to calculate the required reinforcement force. Different formulations of HSM are available depending upon many factors, the equations of equilibrium to be used also being the one [4-9]. Horizontal Slice method has an advantage that it can consider the variation of acceleration along the wall height owing to the horizontal nature of slices rather than vertical alike the conventional methods.

Many researchers have followed the experimental approach to analyze the RSRS using 1-g shake table tests and centrifuge [10-18]. A detailed review on different experimental investigations can be obtained from Sabermahani et al. [19] and Srilatha et al. [20].

The conventional approach to analyze the RSRS is known as Allowable Stress Design (ASD) or more precisely the deterministic approach. This approach increases the Factor of Safety of the structure under consideration to such a high extent that the structure becomes uneconomical. Hence, it has become obsolete and replaced by probabilistic approach which takes into account the uncertainties associated with the parameters involved and a minimum factor of safety is guaranteed just by the satisfaction of limit state function.

The most common approach to perform the probabilistic analysis of RSRS is to calculate the reliability index associated with the slope which gives an idea about its stability. Many researchers have worked on the methodology to find out the surface of minimum reliability index. Li and Lumb [21] and Bhattacharya et al. [22] had found the critical deterministic surface and subsequently that surface was used as the initial surface for investigating the surface of minimum reliability index i.e. β_{min}.

Basha and Babu [23] had used the target reliability based approach (TRA) to analyze the RSRS probabilistically. Three different modes of failure of the reinforcement were chosen for the same and the analysis was done using the first order reliability method (FORM). However, no further validation of the above methodology using another technique such as meta-modelling/ sampling based methods was made. Metya and Bhattacharya [24] had computed reliability of earth slopes using a computational procedure.

The literature mentioned above reveals the fact that there is a dire need to analyze the RSRS probabilistically to account for the uncertainty of geotechnical parameters along with the dynamic nature of the earthquake. The main aim of the present study revolves around the reliability analysis of RSRS using FORM and validating the same using a more efficient and accurate Multivariate Adaptive Regression Splines (MARS) [25] method.

2 Deterministic Analysis of Internal Stability of Geosynthetic RSRS

To analyze the RSRS deterministically, HSM (Fig. 1) is used in the present study in a pseudo-static framework. Soil is assumed to be dry, homogenous and linearly elastic in nature. The backfill is cohesionless and the effect of pore water pressure is neglected. The failure surface is assumed as log-spiral. As discussed in the above section, Nouri et al. [6] had presented a comparison among different formulations of HSM. Hence, simple (*2N+1*) formulation is chosen in the present study which satisfies the horizontal equilibrium for the whole wedge and vertical equilibrium for individual slices together. The number of equations and unknowns are reduced to *2N+1* in this case. The equations of equilibrium used in Shahgholi et al. [4] are used in the same manner to calculate the required tensile strength of the reinforcement (T_j) from Eq. (1).

$$\sum_{j=1}^{m} t_j = T_j = \sum_{i=1}^{N} N_i \sin\alpha + \sum_{i=1}^{N} W_i\, k_h - \sum_{i=1}^{N} S_i \cos\alpha \tag{1}$$

where, S_i = Shear force upon base of slice, N_i = Normal force upon base of slice and W_i = Weight of slice

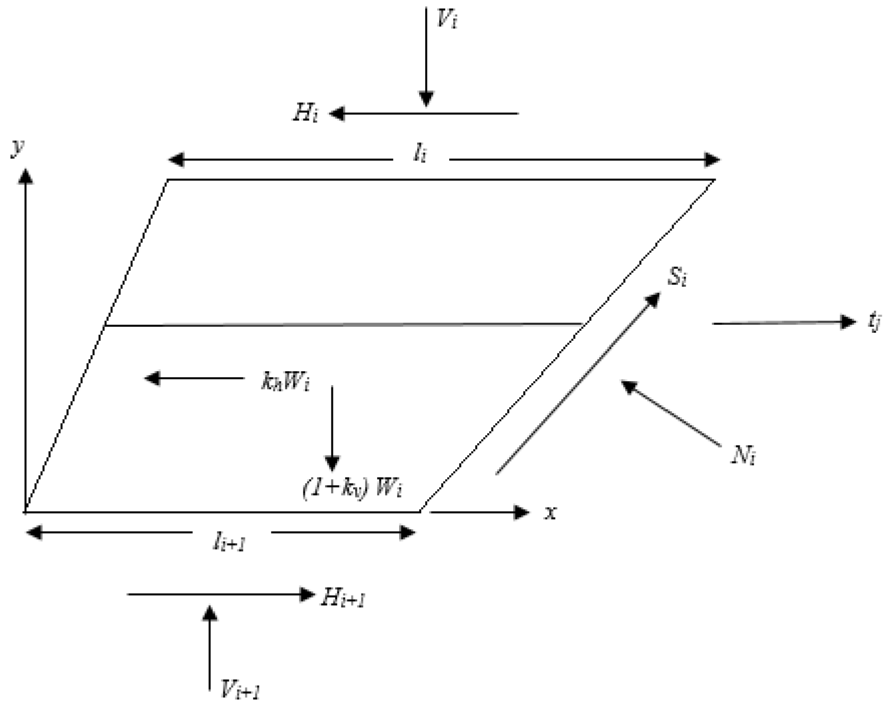

Fig. 1. Forces acting on single horizontal slice with reinforcement.

T_j may be expressed in a non-dimensional term as mentioned below:

$$K = T_j / 0.5\gamma H^2 \tag{2}$$

The value of K is optimized to find out the critical failure surface using Sequential Quadratic Programming (SQP) in MATLAB.

The results from the present study have been validated by comparing them with Ling et al. [2] (Fig. 2) for H =5 m, β = 90^0, γ = 18 kN/m^3, ϕ = 35^0, k_h = 0-0.3 and k_v = 0 and they are in good adherence with the said literature.

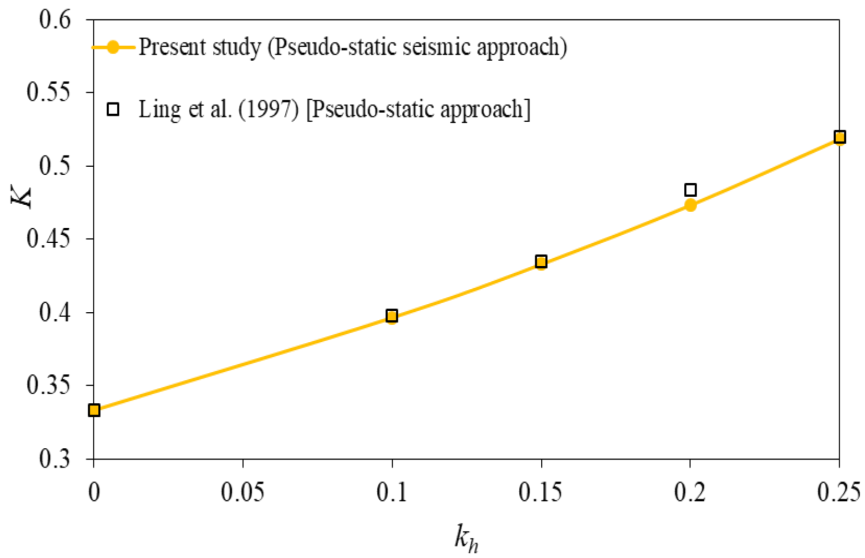

Fig. 2. Comparison of non-dimensional total required tensile strength from the deterministic analysis

3 Probabilistic Analysis

The methodology to calculate the probability of failure (P_f) associated with the critical failure surface is explained in this section. P_f is calculated for two different modes of failure (tension and pullout) followed by the calculation of system probability of failure using FORM and surrogate based Monte-Carlo Simulation (MCS).

3.1 Reliability analysis using FORM

In the present study, the model presented by Metya and Bhattacharya [24] is used to find out the critical probabilistic failure surface and associated P_f. The critical probabilistic failure surface is found out in the same manner as the critical deterministic failure surface i.e.

$$\beta_{min} = min_x \beta\ (G, X) \tag{3}$$

where, β *(G, X)* is the reliability index for a given set of geotechnical parameters (*G*).

The location of the failure surface w.r.t. the origin of the reduced variates determines the reliability or more precisely, the safety of the system. Hence, the required minimum distance may be calculated as follows:

$$D = (X_1{'}^2 + \ldots + X_n{'}^2)^{1/2} \tag{4}$$

where, $X' = (X_1', X_2', \ldots, X_n')$ is a point on the failure surface with minimum distance to the origin.

System reliability index (β_{sys})

In the present study, the series system reliability index is calculated by considering the failure modes to be connected in series. Failing of even one component disables the whole system when the modes are connected in series. P_f for a system connected in series is given by the following equation:

$$P_{fsys} = 1-[\{P_{ft}\}\{P_{fpo}\}] \tag{5}$$

where, P_{ft} = Probability of failure for tension failure mode and P_{fpo} = Probability of failure for pullout failure mode.

Comparison of FORM results

The input variables used in the present study are tabulated in Table 1. All the random variables follow Gaussian distribution except the internal friction angle of soil, which follows the log-normal distribution.

Table 1. Attributes of the random variables used in the present study.

Random variable	Properties		
	Mean (μ)	Coefficient of variation (CoV) %	Distribution
γ (soil unit weight)	18 kN/m^3	5	Normal
ϕ (internal friction angle of the soil)	34^o	5	Log-Normal
T_u (ultimate tensile strength)	20 kN/m	5	Normal
L_{ei} (pullout length of the RF)	0.2 m	0.5	Normal

To validate the formulation, the results from the present study have been compared with Basha and Babu [23] for H =5.5 m, ϕ = 37^0, β = 78.7^0, γ = 18 kN/m^3, k_h = 0.216, N = 10, δ / ϕ = 0.8, T_u = 20 kN/m and k_v = 0. The results are in accordance with the said literature with minor differences noted due to difference in methodologies used.

The value of reliability index (β_t) for tension mode of failure from the present study is 2.806 in comparison to 2.230 from Basha and Babu [23].

3.2 Reliability analysis using MARS based MCS

MARS was primarily developed as a statistical tool to define the relationship between input and output variables [25-27]. MARS approximates the response function based on a forward and backward iterative approach, the main advantage being the automatic identification of basis functions and the parameters associated with them. A MARS model may be formulated as:

$$M = \sum_{q=1}^{n'} \psi_k \, B_q^f \, (z_i) \tag{6}$$

where, ψ is the coefficient of expansion and $B_q^f\,(z_i)$ is the basis function.

The value of generalized cross validation (GCV) defines the goodness of the fit of a MARS model, the best being the one having the lowest value of GCV.

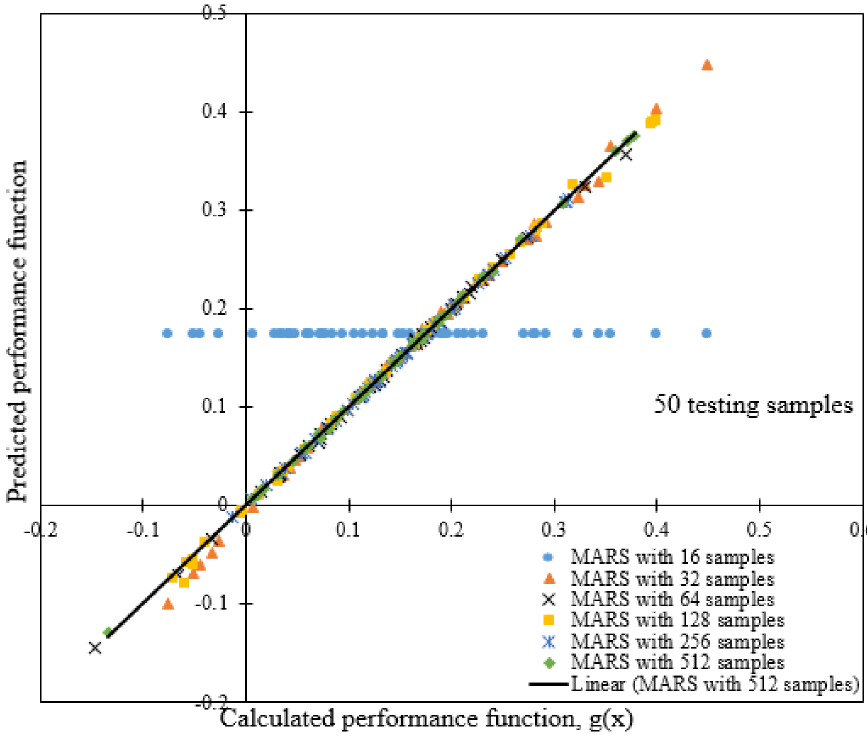

Fig. 3. Comparison between the values of performance function predicted by MARS models and the ones calculated by LEM

To analyze the RSRS using MARS, training data sets were generated using Latin Hypercube sampling (LHS) for the three random variables (T_u, ϕ, γ) used in the study. The value of performance function is obtained using the deterministic code. Further, a group of MARS models were constructed using these data sets where each model consisted of 2^n training samples. Performance of these constructed MARS models is tested using 50 testing samples chosen randomly. The value of coefficient of determination (R^2) is then calculated for each of these models and the optimum number of training samples is noted for the model having maximum value of R^2. The optimum number of simulations is calculated by varying the sample size from 1000 to 80000 and noting the value of simulation after which the fluctuation in P_f is minimal. Moreover, a comparison is drawn between the values of performance function predicted by different MARS models and the ones calculated by the deterministic model as shown in Fig. 3. The results depict very good fitting and also validate the ability of MARS method to predict good results with optimum number of samples taken in the present study. In the present study, the optimum number of training samples for both the modes of failure is chosen as 512, having the maximum value of R^2 equal to 0.99. The optimum number of simulations are chosen as 50000.

4 Results and Discussions

In the contemporary study, RSRS is analyzed using the two methods namely First order reliability method (FORM) and MARS based Monte-Carlo simulations (MCS). The probability of failure (P_f) is calculated for both, tension and pullout modes of failure using the aforementioned methods. Further, system probability of failure is calculated and a comparison is made between the two methods.

4.1 Results using FORM

Probabilistic analysis of RSRS is done using FORM for H =5 m, ϕ = 34^0, β = 90^0, γ = 18 kN/m^3, k_h = 0.2, N = 10, δ / ϕ = 1, T_u = 20 kN/m and k_v = 0. The results have been tabulated below.

Table 2. P_f for different modes of failure using FORM.

Mode of Failure	P_f (%)	β (Reliability Index)
Tension Mode	8.08%	1.40
Pullout Mode	1.16%	2.27
System probability of failure	9.14%	1.33

The results show that the most critical mode of failure is the tension mode having the least value of reliability index as 1.40. However, the probability of failure of system is higher than the tension mode, emphasizing the need to perform system probabilistic analysis.

4.2 Results using MARS based MCS

Probabilistic analysis of RSRS is done using MARS based MCS for same parameters mentioned in section 4.1. The results have been tabulated below.

Table 3. P_f for different modes of failure using MARS based MCS.

Mode of Failure	P_f (%)	β (Reliability Index)
Tension Mode	7.85%	1.41
Pullout Mode	1.39%	2.20
System probability of failure	9.13%	1.33

The results from Table 3 for MARS based MCS are in good accordance with the results obtained using FORM (Table 2). Moreover, the surrogate method follows the same trend as FORM, where tension mode is the critical mode among the two mentioned modes of failure (Fig. 4). The above mentioned results show the ability of MARS based MCS to predict good results. The main advantage of surrogate based modelling is its accurate and efficient performance when the probability of failure is very small, unlike the FORM which suffers from the assumption of linearization of failure surface at the design point.

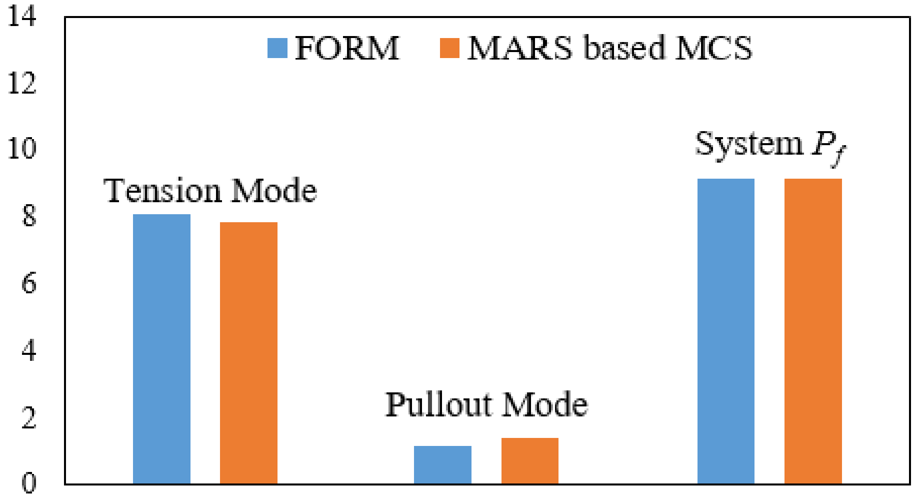

Fig. 4. Comparison between FORM and MARS based MCS

5 Conclusions

In the present study, probabilistic analysis of reinforced soil retaining structure (RSRS) is done in a pseudo-static framework. For this, (*2N+1*) formulation of Horizontal Slice method (HSM) is used. Two different modes of failure are considered for the calculation of reliability indices using FORM. Non-linear programming (SQP) is used to calculate the required tensile strength of the reinforcement. System reliability of failure is calculated by considering the modes of failure to be connected in series. The tension mode of failure is found to be the most critical one, however, the P_f of system is found to be the highest among all. A surrogate based MCS is also performed to validate the results obtained from FORM. Latin Hypercube sampling (LHS) is employed to construct the MARS based model. It is found that MARS based MCS is an efficient, accurate and robust method for the analysis of RSRS and it outperforms FORM in terms of assumptions of the failure surface and computational efficiency. The present study reveals that FORM and MARS based MCS, both are capable of handling the uncertainties associated with the statistical parameters of soil and reinforcement, the latter being more efficient. The probabilistic analysis provides an economical and efficient way of design of RSRS.

Note that, the present study is performed for reinforced soil retaining wall (slope inclination angle > 75^0). For this case, the simple (*2N+1*) formulation provides the most efficient and accurate results [6]. However, as the slope of inclination of RSRS decreases, the results from simple (*2N+1*) incline towards a conservative and uneconomical side due to the non-satisfaction of moment equilibrium equation. Therefore, a more rigorous (*5N-1*) method, which satisfies all the equilibrium equations is recommended for lower inclination angles of RSRS.

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