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Development of Equivalent Lattice Beam Model for Support System in Tunnels and Caverns using FEM Analysis

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Abstract. In Recent years architects and designers make the extensive use of two and three-dimensional lattice structures. The lightweight, ease of construction, freedom of structural shape in lattice structures offer many advantages to the creative designer. Although lattice girder truss elements can model such structures in any finite element package, an equivalent continuum approach is beneficial as in the input of the geometry is much simpler, the number of degrees of freedom reduces drastically, for simple structures like beams or plates, explicit expressions for the deflections and the stress resultants are available in textbooks and the structural behavior of the equivalent continuum model can be understood more easily. The objective of this paper is to use these methods for the determination of equivalent stiffness of rectangular lattice beam, which is a primary rock support system employed in tunnels and caverns. The equivalent stiffness of lattice beam rectangular in shape were determined by four continuum modelling techniques, namely, an equivalent moment of area based on simply supported condition or fixed end condition, method of parallel axis theorem, Burgardt and Cartraud' method and direct energy approach method. The equivalent stiffness so obtained was used as an input parameter in numerical analysis tool STRAND7 for analysis of equivalent rectangular lattice beam. The maximum deflection and bending stress at the centre of equivalent lattice beam were calculated and compared with central deflection and bending stress in case of original lattice girder using this analysis tool STRAND7

Keywords: Equivalent Beam, Continuum Modeling, Tunnel Rock Support.

1 Introduction

In tunneling, Lattice girder poses some similar function as steel arch support, as it serves as an element of temporary lining to support excavation initially and sometime as a part of permanent lining, offer an extremely prudent alternative to other support technique for an extensive variety of ground condition[1][3][4]. Lattice girder is utilized as a NATM support to prevent the development of initial deformation until the point when it is completely cured shotcrete is poured instantly after drilling. It is a combination of truss and beam element and it transfer load to the lower bar through the bending and axial force. Lattice girder has a three dimensional structure and length / width ratios can be 10 to 15. They can be fully integrated with the shotcrete lining and form optimum bond to the ground supported. LG acts as immediate and reliable ground support in the excavation area. The main advantages of lattice girder over conventional steel arch supports are its low weight per meter length of the girder and, a more efficient bond with shotcrete making it economical than other supports [5].

The objective of this paper is to rule out the most reliable method for determination of equivalent stiffness of lattice girder, which is a primary rock support system employed in tunnels and caverns, comparing some techniques available in the literature. The equivalent stiffness of lattice girder were determined by four continuum modelling techniques namely, equivalent moment of area based on simply supported condition or fixed end condition, method of parallel axis theorem, Burgardt and Cartraud' method and direct energy approach method [2]. The equivalent stiffness so obtained was used as input parameter in numerical analysis tool STRAND 7.0 for analysis of equivalent lattice beam. The maximum deflection and bending stress at centre of equivalent lattice beam were calculated and compared with central deflection and bending stress in case of original lattice girder using this analysis tool STRAND 7.0 The results obtained from FEM analysis using four techniques given by researchers are used to determine equivalent stiffness and also used for simulating the results obtained from FEM analysis of lattice girder and equivalent lattice beam. In This paper the design of a simple beam equivalent to a lattice girder has been done and its performance to the actual lattice structure is being compared.

1.1 Continuum Modeling Techniques

The purpose of a computer model is to accurately represent a structure mathematically. However, limitations exist that inhibit the ability to achieve an ''exact'' mathematical model. One such limitation is the number of nodes/members that the computer program can utilize. To help alleviate this problem for structures with trusses, a method has been developed to replace trusses with beam elements thereby reducing the size of the computer model required for analysis. The equivalent beam to a lattice structure (figure -1) shown in figure - 2 is calculated by three basic methodologies in which the equivalent stiffness of the beam is calculated based on the deflection of the loaded lattice girder (3D truss), the equivalent stiffness 'I' is calculated based on the truss cross-sectional area using parallel axis theorem and the equivalent stiffness of the beam is calculated based on strain energy stored in the beam as an action of application of load. The stiffness of continuum lattice beam equivalent to lattice structure is estimated with available techniques proposed by these researchers [2]. The four continuum modeling techniques are equivalent moment of area based on simply supported condition or fixed end condition, method of parallel axis theorem, Burgardt and Cartraud' method and direct energy approach method.

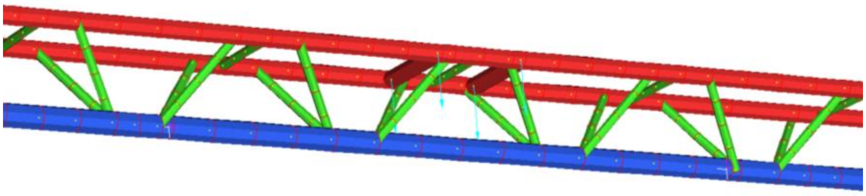

Fig. 1. Lattice girder (LG) under consideration

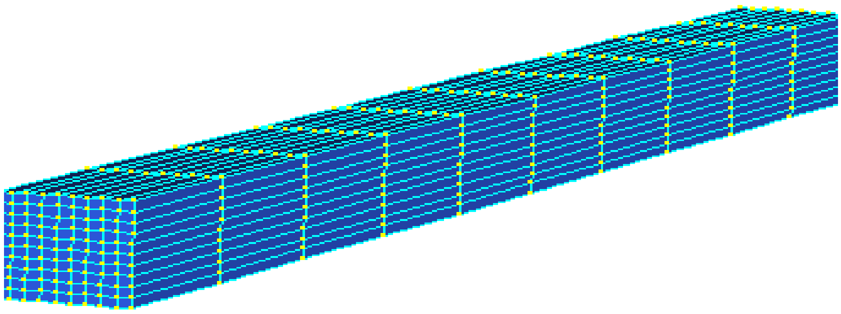

Fig. 2. Equivalent beam

2 Design of Equivalent Beam using Continuum Modeling

The calculation of equivalent properties for beam model using all the four techniques is carried out [6]. The calculation have been carried out for lattice girder LG1: 25x20x10 @170x152.. The Equivalent moment of inertia and area of cross section are calculated for sections LG1 and given in Table 1.

Table 1. Summary of properties of equivalent beams obtained from methods A, B, C and D.

Meth-od	Method Name	Eauivalent beam LG1 25x20x20@100x170	
		MoI, m^4	Side of square beam , mm
A	MOI by Parallel Axis Theorem	2 790 666	78
B	Back analysis from deflection results of FE analysis	1046901	60
C	Burgardt and Cartraud Method	1 914 634	69.3
D	Direct Energy Approach	1 914 634	69.2

2.1 Finite Element Numerical Modeling of Actual Lattice Girder

A finite element analysis program Strand 7.0 was utilized, for the three dimensional numerical testing of the Lattice Girder and equivalent beam. The Strand 7.0 has a user interface to interact with software and for the analysis broadly; two types of models were developed. One type of model was developed to represent the actual lattice girder, and the other types of models were developed to represent equivalent beam. This actual lattice girder model was developed using beam element geometry and the equivalent beam was developed using brick elements. The straight bar components, with circular cross section area of measurements S1, S2, and d, are utilized to represent the lower furthermore, the upper arcing bars, and the diagonal bars of the LG individually. An adequate number of elements are utilized to precisely display the arched geometry of the structure. The standard strengthening steel, grade S500, is utilized in the experiment and the yield stress of the reinforcing steel 500 MPa, Young's modulus 200 GPa, Poisson's proportion 0.3, is used for the inelastic investigations.

Kinematics constraints were provided at both the end of support of the lattice girder model. The left end support of LG Model, pin support was provided only z rotation allowed and translation was restrained in all directions while on the right end of LG Model, roller support was provided only x translation, and in plane z rotation were allowed. The attribute were applied along the chord length of the LG arc model, and deflection was measured at center of lower bar and plotted load versus deflection curve and compared with the experimental data. The material property of reinforcing bar used in numerical analysis and sectional properties of lattice girder is shown in Table 2.

Table 2. Material properties for analytical evaluation & sectional properties

		Elastic modulus (MPa)	Unit weight (kN/m^3)	Poisson ratio	MOI (mm^4)	Cen-troid (mm)
Triangu-lar		200,00 0	7,850	0.3	3219.49 7	67.02
Lattice girder	Upper, Diagonal, and Lower chord					

FEM Analysis of Equivalent Lattice Beam

The dimensions of the equivalent lattice beam being estimated using four continuum techniques. The equivalent lattice beam shape assumed square, and mate rial was chosen as mild steel. The Material properties & sectional properties for numerical evaluation are listed in Table 3.

Table 3. Material properties for analytical evaluation & sectional properties Lattice Beam (Square shape)

Method used for estimating equivalent stiffness	Size of square Beam (mm)	Elastic modulus (MPa)	Unit weight (kN/m3)	Poisson ratio
Method-A : Equivalent moment of inertia by Parallel Axis Theorem	78.2	200,000	7,850	0.3
Method-B : Deflection of The Loaded Lattice Girder	60	200,000	7,850	0.3
Method-C :Burgardt en Method	69.24	200,000	7,850	0.3
Method-D : Direct Energy Approach	69.24	200,000	7,850	0.3

Numerical modeling of equivalent beam was done using the finite element method (FEM) based software STRAND7.0. The geometry of equivalent beam created, material and sectional properties are assigned to beam.

The vertical downward concentrated point load on Centre of lattice beam applied till yield point of beam. Kinematics constraints were provided at both the end of support of the lattice girder model. The left end support of LG Model , pin support was provided only z rotation restrain and translation was restrained in all direction while on the right end of LG Model, roller support was allowed only x translation and in plane z rotation restrained. Attribute were applied along the chord length of LG arc model. The maximum central deflections of beam in vertical and transverse directions were measured. The contour diagram of deflections in x and y directions are generated using the numerical tool.

All the results from analysis of numerical models of actual lattice girder and equivalent lattice girder obtained by all four methods are summarized in the table 4. This Summary enlists calculated value of Moment of inertia, value of vertical displacement corresponding to the load of 40 KN.

The calculations have been carried out for lattice girders , LG 1 and equivalent beam and deflection results are shown in Table 4.

Table 4. Summary of central deflections in lattice girder and equivalent beam

S no.	Method of analysis of Equivalent Beam	LG 1 : MoI,mm^4	Load, kN	Mid-point vertical displacement,mm
1	**Actual LG**	**2 790 666**	**40**	**3.98**
2	**Four Methods (Equivalent LG Beam.)**			
A	Moment of Inertia Method	2 790 666	40	2.005
B	Back analysis using standard formula	1 046 901	40	3.98

	(Using value of deflection for back analysis from actual numerical model)			
C	Burgardt and Cartraud	1 914 634	40	3.356
D	Direct Energy Approach	1 914 634	40	3.356

3 Conclusions

1. The observations from the table 4 shows that vertical deflections derived from FEM analysis of lattice girder structure and equivalent lattice girder beam by four methods are comparable and Method –B gives exact simulating result.
2. The Method-A is least accurate and Method C and D gives similar deflection values. The techniques given by Method C and method D shows the results second best close to results of numerical analysis.
3. The size of equivalent lattice beam has been designed and dimensions of beam shows that technique given by method B: Back analysis method provides the most conservative cross section of the beam. Similarly, the method A: Moment of inertia by parallel axis theorem method estimates the most economical cross section of the beam

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