

Application of Bi-Directional Extreme Learning Machine in Predicting Stability of Slope of Railway Embankment in Seismic Condition

Abi Bardhan¹, Dhilipkumar B², Lakshmi Mulagala³ and Pijush Samui⁴

¹ Researcher Scholar, Department of Civil Engineering, National Institute of Technology Patna, Patna-800005, India. E-mail: abidhan@nitp.ac.in

² Senior Research Fellow, Department of Civil Engineering, National Institute of Technology Patna, Patna-800005, India. E-mail: dhilipkaan@gmail.com

³ Geotechnical Expert, NK Consortium (NK-OC-JTC-NKI-QCI-RITES), PMC-DFCCIL New Delhi, India. E-mail: m.lakshmi.p3@nkindia.in

⁴ Associate Professor, Department of Civil Engineering, National Institute of Technology Patna, Patna-800005, India. E-mail: pijush@nitp.ac.in

Abstract. This study presents the application of machine learning technique in predicting the stability of slope of Railway embankment. As we know that most of the railway tracks are built on the natural ground surface and the track system is installed on the prepared formation called Permanent Way, constructed either earthwork in formation or earthwork in cutting or in a combination of both. In contrast, the installation of a railway system requires healthy investment as well. Hence, to run the entire system satisfactorily a detailed analysis on account of stability of slopes of Permanent Way is always necessary specially to prevent the damages in seismic conditions. For this purpose, Bi-directional Extreme Learning Machine (Bi-ELM) is used for the prediction of Factor of Safety in seismic conditions. It is interpreted from the results that the developed model is capable enough to predict the factor of safety. The value of R^2 obtained as 0.9983 in both the training and testing phases. The lower values of RMSE (0.0184 in the training phase and 0.0181 in the testing phase) in both cases justify the generalization capability of the model. The finding of this research concludes that the developed model can be used as a simple computational approach in predicting the stability of slopes particularly in introductory stages of a Railway Project.

Keywords: Slope Stability Analysis; Railway Embankment; Artificial Intelligence; Machine Learning; Extreme Learning Machine.

1 Introduction

Railway tracks are normally built on the natural ground surface or the elevated formation and the tracks are laid on a prepared bed called Permanent Way, prepared either in cutting, filling of earthwork, or combination of both. Necessary longitudinal

gradient and cross slope are provided during the construction of the formation bed. The rails are attached to the sleeper with help of fasteners and the sleepers rest on ballast cushion. All these elements together are called 'rail track system' and act as a single unit. In general railway projects are carried out with large earthwork which leads to high initial expenditure during the course of the construction of permanent way. Therefore, for the safety of the entire system, a detailed analysis should be done before the track system comes into actual use.

Evaluation of slope stability is a challenging task for geotechnical engineers which is the most encountered problem in the transport network. The heterogeneous property of soil makes it difficult for engineers to assess the reliability accurately. Nowadays several methods are available to determine the slope stability of the embankment. These methods are two types i.e. deterministic or probabilistic [1]-[4]. Methods like Strength reduction method (SRM), Limit equilibrium method (LEM), limit analysis method comes under deterministic type. The deterministic analysis is based on the evaluation of the factor of safety which will determine the stability of the embankment. Among the various slip surfaces, the one which gives a minimum factor of safety is termed as a critical slip surface and the corresponding factor of safety is termed as critical factor of safety. For the determination of the critical factor of safety trial and error approach is normally being used.

On the other hand, undesirable constituents such as pebbles, wastes, organic matter, etc. develop spatial variability within the soil and uncertainties in soil parameters i.e. cohesion, internal friction, unit weight. Because of these uncertainties involved, the deterministic methods have limitations in their application. Using Limit Equilibrium Method (LEM) satisfactory results can only be obtained when the soil parameters are correctly accessed. Therefore, due to the complex and multi-factorial interactions between factors that affect slope stability, the task of assessment of slope stability remains a significant challenge for geotechnical engineers [6]. Considering these phenomena, this study implements a soft computing approach i.e. Bi-Directional Extreme Learning machine for the assessment of slope stability. The remainder of this paper is structured as follows including the Instruction section. In the next section, the methodological details of LEM techniques along with proposed soft computing techniques i.e. ELM and Bi-ELM are furnished. This is followed by, discussion of the analysis of slope stability and results and discussion. At the end, a summary and conclusion are furnished.

2 Methodology

2.1 Conventional analysis of slopes

In LEM technique, factor of safety of slopes can be defined as the ratio between resistance and disturbance along a probable slope surface. Many available methods that

are based on the method of slices [5] can be used to determine the factor of safety. For a typical slope given in Fig. 1, the factor of safety can be determined using the Bishop's simplified method given by,

$$FOS = \frac{\sum_{i=1}^n [C_i \Delta x_i + (W_i - u_i \Delta x_i) \tan(\phi_i)] \frac{\sec \alpha_i}{1 + \tan(\phi_i) \tan(\alpha_i) / FOS}}{\sum_{i=1}^n (W_i \tan \alpha_i)} \quad (1)$$

where W_i are Δx_i are the weight and width of the i th slice respectively, ϕ_i and C_i represent the angle of internal friction and cohesion respectively at the base of i th slice, u_i is the pore water pressure and α_i is the tangential angle respectively at the base of i th slice and n denotes the total number of the slice. To calculate the critical factor of safety trial and error method are used.

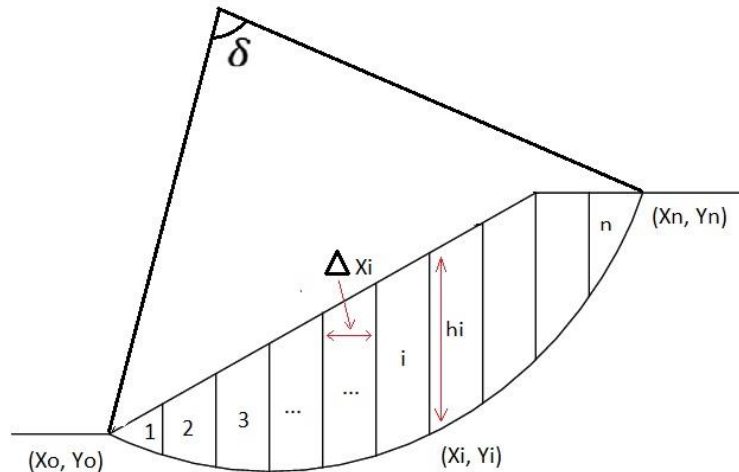


Fig.1. Sectional details of a typical soil slope

2.2 Extreme Learning Machine

Extreme learning machine i.e. ELM, proposed by Huang et al. [6] is a type of single layer feed-forward network used in classification and regression problem. During training, weights and biases are assigned at random and then the output weights are calculated with defined number of the hidden layer neurons and activation function. Network structure of ELM is shown in Fig.1 with a single hidden layer having 2 neurons.

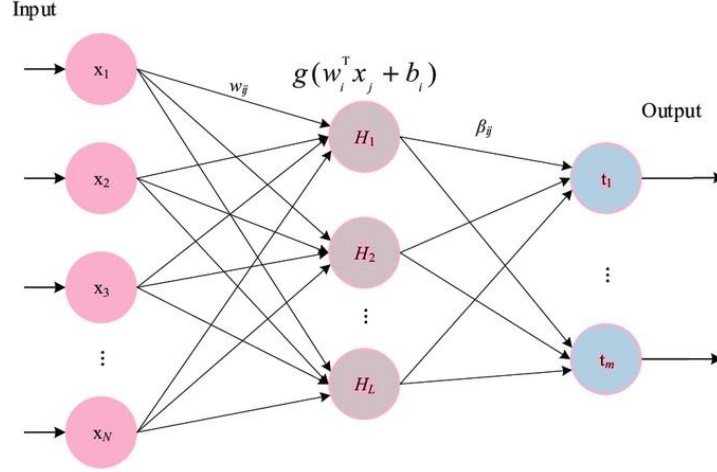


Fig.1. A structure of ELM showing a single hidden layer with 2 neurons

Now, for example, a set with input $X_i = [X_{i1}, X_{i2}, X_{i3} \dots, X_{in}]^T \in \mathbb{R}^n$ and output $Y_i = [Y_{i1}, Y_{i2}, Y_{i3} \dots, Y_{im}]^T \in \mathbb{R}^m$ where $i = 1, 2, 3, \dots, p$, then the structure of the ELM can be mathematically expressed as:

$$t_j = \sum_{k=1}^p \beta_k g(w_k, b_k, z_j) \quad (2)$$

where w_k and b_k represent the input weight and biases of the k th hidden node, $z_j = [z_{1j}, z_{2j}, z_{3j} \dots, z_{nj}]^T$ is the output weight, $g(w_k, b_k, z_j)$ represents the output of the k th hidden node to the input of z_j , t_j represents the predicted output of the corresponding input z_j and j is the number of training sample. Right after the generation of input weight and biases randomly, ELM estimates β using the following expression:

$$\beta_{min} = ||H \times \beta - T|| \quad (3)$$

And, in the next step, the output weights are calculated using linear equation given by,

$$\beta_{min} = H * pinv(Y) \quad (4)$$

where H represents the output vector coming from hidden layer and $pinv(Y)$ represents the Moore-Penrose pseudoinverse of training data. The output vector can be written as:

$$H = \begin{bmatrix} g(w_1, b_1, x_1) & \dots & g(w_p, b_p, x_1) \\ \vdots & \ddots & \vdots \\ g(w_1, b_1, x_q) & \dots & g(w_p, b_p, x_q) \end{bmatrix} \quad (5)$$

Lastly, the output weight from the hidden layer is used to get the desired output for a new set of data i.e. testing data as follows:

$$Y_{i-new} = X_{i-new} * Z_j \quad (6)$$

2.3 Bi-directional Extreme Learning Machine

Although the structure of ELM seems simple and has much faster training speed than traditional tuning-based learning methods, but, the selection of optimum number of neurons in the hidden layer still remain intractable challenges. Generally, the number of hidden neurons is predefined by the users and the trial and error approach has been followed to obtain the optimal no. of hidden neurons. Thus, this process can't always obtain the best network structure which likely causes under-fitting or over-fitting.

To avoid this problem, Yang et al. [7] proposed Bi-Directional Extreme Learning Machine (Bi-ELM), in which Yang divides the training operation into two parts. When the number of hidden nodes $L \in \{2n + 1, n \in Z\}$, the hidden node parameter (ω_i, b_i) is generated randomly. When the number of hidden nodes $L \in \{2n, n \in Z\}$, the hidden node parameter (ω_i, b_i) is obtained as per the following expressions:

$$\hat{\omega}_{2n} = g^{-1}(u(H_{2n})) \cdot x^{-1} \quad (7)$$

$$\hat{b}_{2n} = \sqrt{\text{mse}(g^{-1}(u(H_{2n})) - \omega_{2n} \cdot x)} \quad (8)$$

$$\hat{H}_{2n} = u^{-1}(g^{-1}\omega_{2n} \cdot x + b_{2n}) \quad (9)$$

where u^{-1} and g^{-1} indicate the inverse functions of u and g , respectively.

Several applications of Extreme Learning Machine are available in the literature [8]-[11] and researchers are using this technique in predicting the desired output in every field of engineering.

3 Slope Stability Analysis

As mention in the previous section, limit equilibrium approach is used in this study in estimating the stability of slopes. To perform the analysis, 50 data sets are generated randomly considering the lower limit and upper limit of the soil parameter (Table 1) and then the slope stability analysis has been performed in SLOPE-W software separately for static as well as in seismic condition. A 12.293 m high embankment with 2H:1V side slopes are considered in the analysis. The shear strength properties for different layers are presented in Fig.2. The typical section of the proposed formation comprises

of sleeper and ballast cushion (350 mm depth) at the top followed by a 600 mm thick blanket layer and which is underlaid by 1000 mm thick prepared subgrade. The prepared subgrade is an integral part of the embankment fill of varying height.

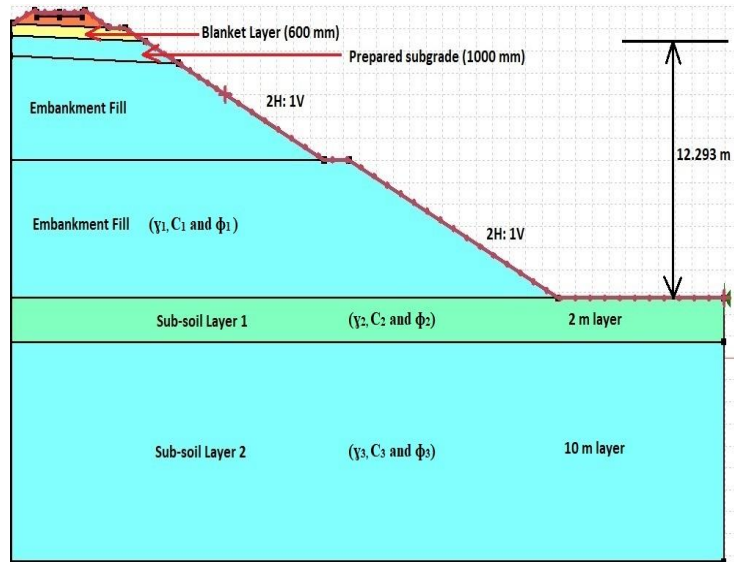


Fig.2. Cross-sectional details of 12.293 m high embankment

For the embankment fill, the values of shear parameters considered as γ_1, C_1 and ϕ_1 , for the sub-soil layer-1 it is γ_2, C_2 and ϕ_2 and for sub-soil layer-2 it is γ_3, C_3 and ϕ_3 . The statistical details of the shear parameters of different layers considered in the analysis are given in Table 1.

Table 1. Statistical details of soil parameters for different layers

Parameters	Min	Max	Mean	Median	Std. Error	Std. Dev.	Variance	Kurtosis	Skewness
γ_1	17.00	18.43	17.76	17.78	0.05	0.46	0.21	-1.47	-0.02
C_1	2.00	10.00	5.70	5.68	0.22	2.23	4.97	-1.09	0.08
ϕ_1	26.00	33.70	29.67	29.75	0.23	2.34	5.50	-1.26	0.07
γ_2	15.50	18.60	16.96	17.11	0.08	0.80	0.64	-0.83	-0.07
C_2	0.00	155.00	83.88	91.00	4.41	44.06	1941.52	-0.90	-0.37
ϕ_2	9.80	34.00	21.70	21.05	0.74	7.40	54.83	-1.18	0.12
γ_3	16.00	17.70	16.88	16.89	0.05	0.47	0.22	-1.10	-0.09
C_3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-	-
ϕ_3	27.00	31.00	28.99	28.96	0.10	1.03	1.05	-0.76	-0.03

4. Results and Discussion

4.1 Evaluation of FOS in static and seismic conditions

The values of factor of safety have been calculated in SLOPE-W software in static conditions as well as seismic conditions. To determine the factor of safety in seismic conditions, the values of co-efficient of ground acceleration k_h and k_v are considered as $0.16g$ and $0.08g$ respectively for Zone III. The values of Z, I and S are taken as 0.16, 1.5 and 2.0. Fig. 4 represents the variation of factor of safety for different values of cohesion, angle of internal friction and unit weight of soils. The values of critical factor of safety in static and seismic conditions are obtained as 1.622 and 1.146 respectively. Fig. 4 and Fig. 5 represent the critical failure surface respectively for static analysis and seismic based analysis.

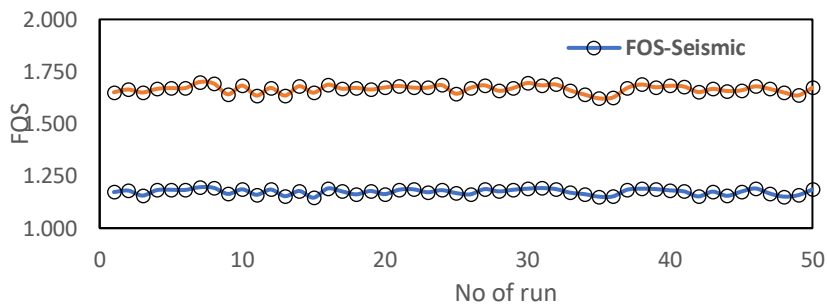


Fig. 3. Variation in FOS in Static and Seismic Condition in different cases

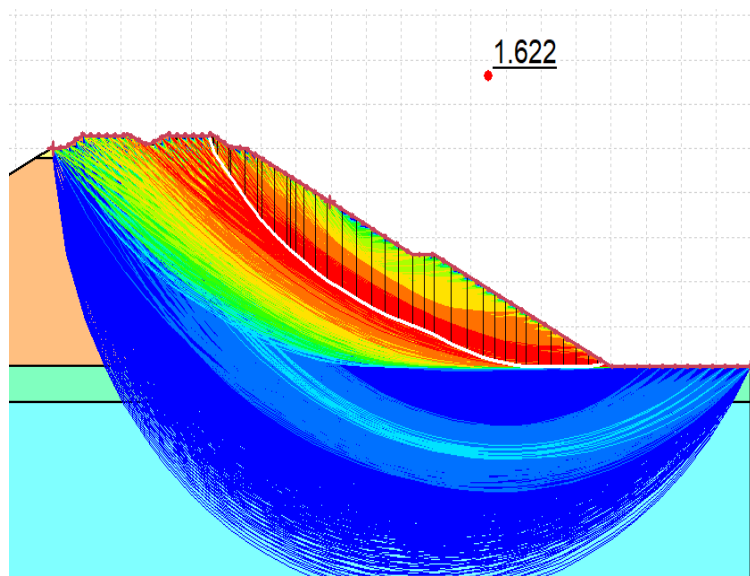


Fig. 4. Analysis of factor of safety in static condition showing all failure surface

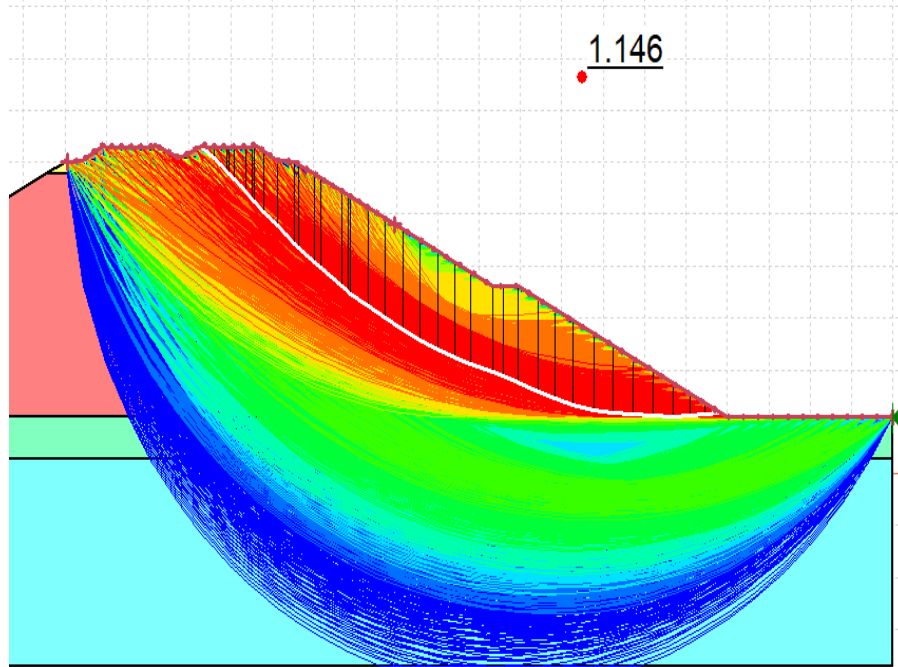


Fig. 5. Analysis of factor of safety in seismic condition showing all failure surface

3.1 Assessment of Bi-directional ELM Model

Now, prior to the development of the model, the total dataset has been normalized between 0 and 1, and then divided into two portions, i.e. training and testing. In the training portion, 75% of the entire dataset has been selected randomly and the balance 25% data is considered as a testing dataset. The training dataset is used to develop the Bi-ELM model while the testing dataset is used to validate the developed model. Once the model is developed, the capability of the developed model is then assessed using five performance parameters namely, Adjusted R2 ($Adj.R^2$), Mean Absolute Error(MAE), determination co-efficient (R^2), Root-mean square error (RMSE) and Weighted Mean Absolute percentage error (WMAPE), which can be mathematically expressed as:

$$Adj.R^2 = 1 - \frac{(n-1)}{(n-p-1)}(1-R^2) \quad (10)$$

$$MAE = \frac{1}{n} \sum_{i=1}^n |\hat{y}_i - a_i| \quad (11)$$

$$R^2 = \frac{\sum_{i=1}^n (a_i - a_{mean})^2 - \sum_{i=1}^n (a_i - p_i)^2}{\sum_{i=1}^n (a_i - a_{mean})^2} \quad (12)$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (a_i - p_i)^2} \quad (13)$$

$$WMAPE = \frac{\sum_{i=1}^n \left| \frac{a_i - p_i}{y_i} \right| \times a_i}{\sum_{i=1}^n a_i} \quad (14)$$

where, a_i and p_i are the observed and predicted i th value, n is the number of samples in a dataset, a_{mean} is the mean of the observed values. For a perfect model, value of these statistical parameters should be equal to their ideal value as $Adj. R^2 = 1$, $MAE = 0$, $R^2 = 1$, $RMSE = 0$ and $WMAPE = 0$.

Table 2 represents the details of performance parameters determined for the developed model in the training and testing phase. As can be seen, the values of all the performance parameters are quite close to their ideal value. The Bi-ELM model achieved 100% accuracy in terms of R^2 value ($R^2=0.9983 = 99.83\%$) in the training phase as well as in the testing phase. Also, the values of MAE (0.0149 in training and 0.0150 in testing phase) and RMSE (0.0184 in training and 0.0181 in testing phase) show that the model has very good generalization capability in both the phase. It is also understood from the other performance parameters that the capability in predicting the factor of safety in static condition as well as in seismic condition of the developed model is quite satisfactory.

Table 2. Performance parameters of the developed model in training and testing phase

Parameters	Bi-ELM (Training)	Bi-ELM (Testing)
Adj. R^2	0.9980	0.9971
MAE	0.0149	0.0150
R^2	0.9983	0.9983
RMSE	0.0184	0.0181
WMAPE	0.0312	0.0259

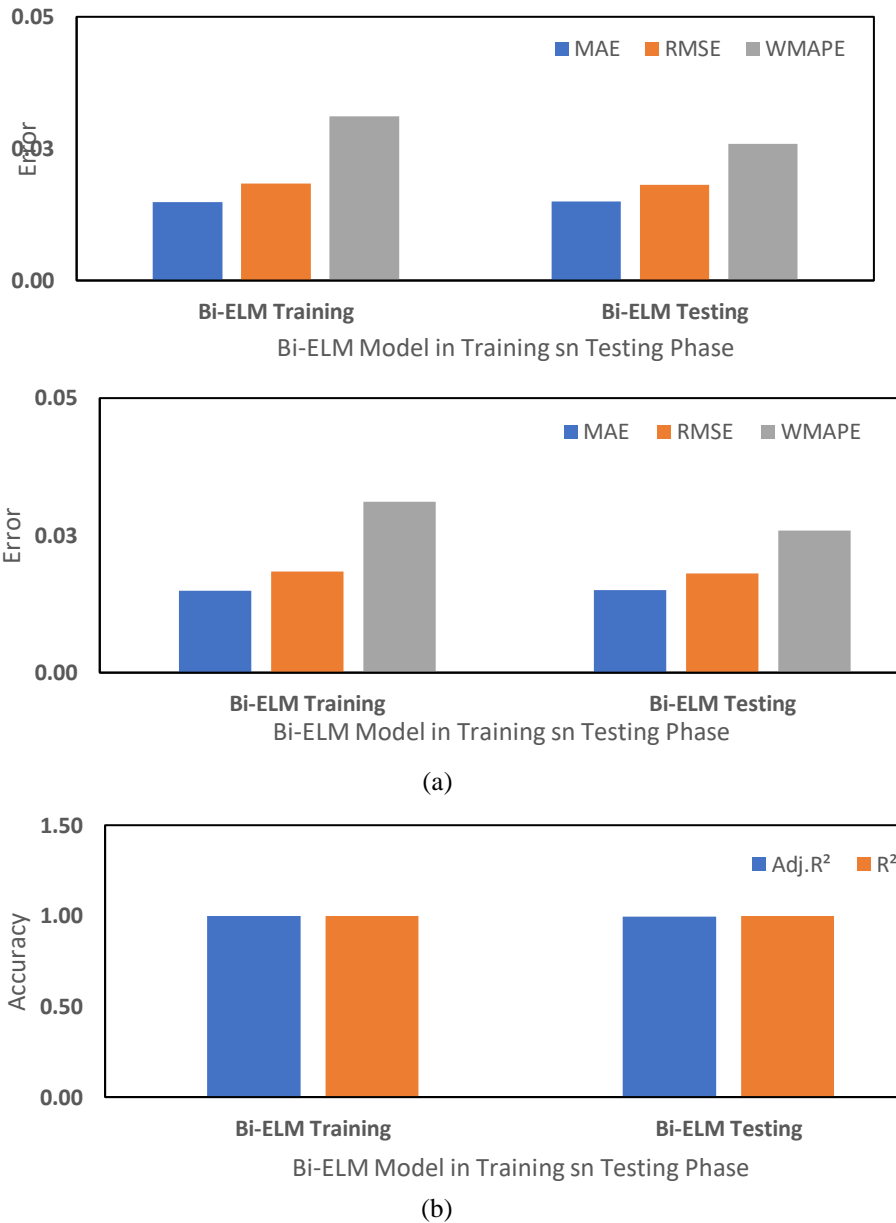


Fig. 6. Graphical Representation of the Performance of the Bi-ELM Model Fig. a) showing MAE, RMSE and WMAPE Model Fig. b) showing Adj. R² and R²

Also, the match between the actual and predicted values of factor of safety is shown in Fig.8 in the form of a scatter plot. As can be seen, all the values predicted by Bi-ELM is lies on the line of ideal model and hence the developed Bi-ELM model can be

considered as a perfect model to predict the factor of safety of any railway embankment provided that the wide range of data is available.

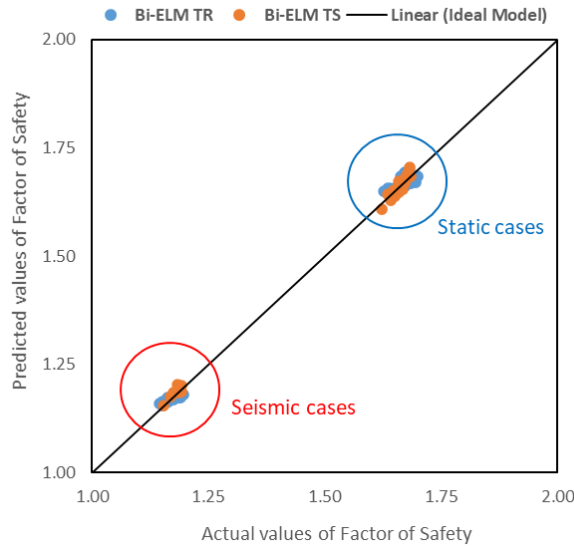


Fig. 7. Scatter plot showing the match between actual and predicted values of factor safety.

4 Summary and Conclusion

In this study, the authors developed a soft computing model i.e. Bi-Directional Extreme Learning Machine to predict the slope stability of railway embankment. For this, the values of shear strength parameters of soils are generated randomly and considered in the analysis. A 12.293 m high embankment is considered with embankment fill, sub-soil layer 1 and sub-soil layer 2. Different values of cohesion, angle of internal friction and unit weight are considered for all three layers. A set of 50 data generated randomly and assigned in each run to obtained the factor of safety in the static case and well as in the seismic case separately.

In the next stage, the values of $\gamma_1, C_1, \phi_1, \gamma_2, C_2, \phi_2, \gamma_3, C_3, \phi_3, k_h$ and f_{os} is used to develop the soft computing model. However, prior to the development of the model, the entire dataset has been normalized between 0 and 1 and then divided into training and testing datasets. Right after the data partitioning, the training dataset is used to develop the Bi-ELM model. Later, the developed model is assessed in terms of performance parameters ($Adj. R^2$, MAE, R^2 , RMSE and WMAPE). It is understood from the results of the performance parameters that the developed model is capable enough in predicting the factor of safety. Overall, the Bi-ELM model is highly

recommended as an intelligent tool to assist in decision-making process for the assessment of slope stability.

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