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Genetic Algorithm based Optimization and Design of Pile Foundation

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Abstract. Piles are deep foundations of relatively small diameter shaft, introduced into the soil by suitable means, to support the load coming on it from the superstructure when a good bearing stratum is not available near the ground surface. This study attempts an optimized design approach for single piles in order to evaluate the pile geometry, based on the ultimate bearing capacity of the pile within stipulated limits. A standalone software code has been developed as a result of this study. The code is written in MATLAB that incorporates Genetic Algorithm (GA) optimization technique to obtain optimum pile dimensions depending upon supporting soil condition and incoming loads. The pile is then designed satisfying all the structural requirements as per Indian Standards Code for Reinforced Concrete cast-in-situ pile, IS: 2911. The optimization and design procedure is demonstrated through the proposed software and the results are verified manually. The optimum pile dimensions, reinforcement details and an estimate of the design are presented as the software output.

Keywords: computer aided design of pile foundations; deep foundation; GA; MATLAB software; optimization algorithm.

1 Introduction

Piles are deep foundations of relatively smaller diameter shaft which are driven into the ground or introduced into the soil by suitable means so as to support the load coming on it from the superstructure when a good bearing stratum is not available near the ground surface or at shallow depths. In such situations load have to be transmitted to a firm strata capable of supporting such loads even though such strata may be at an appreciable depth below the ground surface. Piles are used in a wide variety of important structures and are not only subjected to vertical loads coming from the superstructure, but also to lateral loads coming from inclined loads, wind, waves, earthquakes, uplift forces etc. [22].

Indian Standards [9],[10],[11],[12] classifies Concrete Piles as Driven Cast-In situ piles, Bored Cast In-situ piles, Driven Precast piles and Precast piles in Pre-bored

Holes. The capacity of the pile foundation is dependent on the material and geometry of each pile, their spacing in pile group, the load bearing strength and type of the surrounding soil supporting the pile, the method of pile installation, and the direction of applied loading[1]. The soil bearing capacity is affected by many factors such as type and strength of soil, foundation dimensions, soil weight, surcharge, type of loading etc. [21] . The variability of the properties of the supporting soil, soil profiles and multi-layered soil profiles, which generally exists in nature, affects the load bearing capacity of the pile drastically. The choice of a particular pile is governed by certain situations, namely, site conditions, economy, time considerations etc. Hence, in order to encounter such problems with large variability, the aid of modern high-speed computers and software has become popular without which this job would be tedious.

Conventional design approach for piles requires determining the load carrying capacity of piles which can be obtained from Dynamic Formulae, Static Formulae, Pile load test or Penetration Test depending upon a particular situation giving due consideration to soil exploration data and chemical properties of soil. Depending upon the load coming from the superstructure the pile or group of piles are designed in such a way so as to transfer the load to the supporting soil safely. In geotechnical design part, the Ultimate Load Carrying Capacity and the Allowable Load Carrying Capacity of the pile must be checked whereas in the structural design part the designed foundation must be checked for bearing capacity, driving and handling stresses. Foundation designs are usually a trial-and-error procedure, in which a trial design (foundation dimensions and reinforcements) is chosen and is checked against the geotechnical and structural requirements, which is followed by revision of the trial design, if necessary [15]. But due to various assumptions and approximations associated with such method of estimation of load bearing capacity and design methodology, the pile design often comes out to be conservative. A conservative design may lead to an uneconomical design. An optimization approach in the foundation design process may confirm the economic design. Hence, an optimised design methodology is the need of the hour. Optimized design approach for single piles evaluate the Ultimate Bearing Capacity of the pile within stipulated limits and changing the pile dimensions iteratively until a reasonable pile geometry is found to support the total load coming on it. The pile is then designed structurally for bearing the loads acting on the pile satisfying all the structural requirements.

The research done in foundation engineering for the application of optimization methods using Computer Aided Design methods is scarce. In particular, researches dealing with pile optimization assume different ideas mainly with groups of piles and simplification of the structural capabilities of reinforcement which often have limited practical considerations. Chow and Thevendran [5] used pile length as the main design variable to minimize differences in bearing loads between the piles in the pile group. Hoback and Truman [7] introduced a weightless optimality rule into the original optimality criteria approach to treat design variables, (e.g., the spacing and battering of the piles) that has no measurable effect on the objective function. Huang and Hinduja [8] adopted a quasi-Newton method to optimize the shape of a pile foundation with the assumption of a linear force-deflection relationship for the pile-soil system. Valliappan et al. [27] applied the generalized reduced gradient method to optimize pile foundation design with the lowest cost objective. Their design variables included pile length, diameter, number and pile cap. The allowable total and differential settlements were the only constraints. Kim et al. [16],[17] used GA and recursive quadratic programming respectively to optimize the layout of a pile foundation, with minimum differential settlement being the objective and with the assumption of linear pile-soil interaction. Chan et al. [4] presented an automatic optimal design method using a hybrid GA for pile group foundation design with the concrete volume of the piles and the cap as the objective function. Letsios et al. [19] proposed the formulation of an optimization problem using German Foundation Code DIN 4014 and Euro code 7 (EC7) design procedure. Both standards are based mainly, on four pile design criteria: (i) Axial bearing Capacity, (ii) Acceptable settlements, (iii) Strength of pile as a structural element and (iv) Lateral bearing capacity and acceptable horizontal displacements. Pérez et al.[22] dealt with the optimization of three different materials for single piles: steel pile, concrete pile, and steel fibre reinforced concrete pile by developing a program using interior point algorithm implemented in MATLAB and an objective function based on the cost for each of the pile materials. Darius et al. [6] presented optimization of grillage foundation using GAs for integrating MATLAB environments with the goal of optimization so as to obtain the optimal layout of pile placement in the grillages without taking cost into account. Islam and Rokonuzzaman [15] used CAD to demonstrate design of foundation which is done in two different stages: geotechnical design and the structural design. Most of the design concentrated on bearing capacity and settlement criteria ignoring the economy of the design. Nikolaou and Pitilakis [18] developed a stand-alone program based on MATLAB for the calculation of bearing capacity and settlements of shallow foundations using several well-known formulas from the literature and design codes that are preferred in engineering practice

All the above research has contributed greatly to the problems of optimizing pile foundation design. However, most of these methods might be difficult to apply in practice owing to the fact that the adopted design method and constraints are not always code-based , Also the objective function fails to consider all the variables involved in pile foundation design and construction. In many cases practical considerations or limits are not considered in design which causes problems in practical implementations. These shortcomings and the wide gap between conceptual optimization and design against the actual field applicability, ultimately, fail to cater to the needs of a designer.

This study focuses on bridging such gaps in conceptual design and practical applications by optimizing pile foundation design using a GA and Indian Standards code [9],[10],[11],[12],[13],[14] specifications. The GA is used to optimize the pile dimensions based on Indian Standards [9],[10],[11],[12] methodology within recommended limits and user defined practical specifications. A standalone software code has been developed using MATLAB programming for automating the entire optimization and design process. An estimate [3] for construction is also presented which may further ease the choice of selection of a particular design.

This study primarily makes an attempt to develop a GA for Geotechnical Design optimization of Pile Foundation for a given set of soil and load parameters.

2 Methodology

2.1 General

This study makes an attempt to develop a GA for optimization and design of a pile foundation. The Optimization methodology used in optimization in the said software is GA. GA is a search-based optimization technique that works on the principles of Genetics and Natural Selection which is used to find optimal solutions to difficult problems which otherwise would take a considerable amount of time to solve [2]. It is frequently used to solve optimization problems particularly in research. The Pile Design, both geotechnical and structural, is as per Indian standards code [9],[10],[11],[12],[13],[14] based design method. A preliminary estimate of conceptual design is also presented which is as per Assam Public Works Department Schedule of Rates [3]. Popular programming software MATLAB has been used to design the interface of the said software and link the relevant codes for optimization and design to the interface.

2.2 Genetic Algorithm(GA)

The GA is a stochastic search-based optimization algorithm inspired by the process of evolution by means of natural selection, primarily fit for nonlinear optimization problems, where gradient based methods cannot be applied due to lack of smoothness of the objective function [2] .The algorithm makes use of processes such as selection, crossover, and mutation to improve upon a set of solutions and converge towards an optimal solution. Each solution consists of a set of properties, and it is through manipulation of these properties that the GA can converge towards a good solution. In GA, a population is a group of solutions, also referred to as individuals. For a given optimization problem, a group of potential solutions is initially generated, i.e. the first generation. Within a population, each solution or individual has a corresponding fitness value calculated with a fitness function. The fitness value indicates how good, or fit, a solution is. Within each population, some numbers of the fit solutions are chosen to carry over to the next generation of solutions as clones. These solutions guarantee that the best fitness value of each generation will either be maintained from one generation to another or improved upon, while providing properties of higher quality within a population.

Crossover is a reproduction function. Within each generation, a smaller group of solutions is selected to combine their properties in order to create new solutions, and these new solutions are considered a new generation. The crossover group is called parents, and the "genes" of two (or more) such parents are combined in a randomized way to produce one (or more) new solution, their child. It is these children that make up a new generation, and since parent solutions mostly have good fitness values, the new generation is expected to be better in terms of fitness [2].

Each solution may be subject to mutations; random changes in one or more properties (genes) of a solution according to some probability, referred to as the mutation rate [2]. Mutations exist in order to maintain and introduce diversity into a population. With more diversity, the algorithm has a lower risk of ending up in suboptimal local minima. A flowchart of the simple, general version of GA is shown in Fig. 1

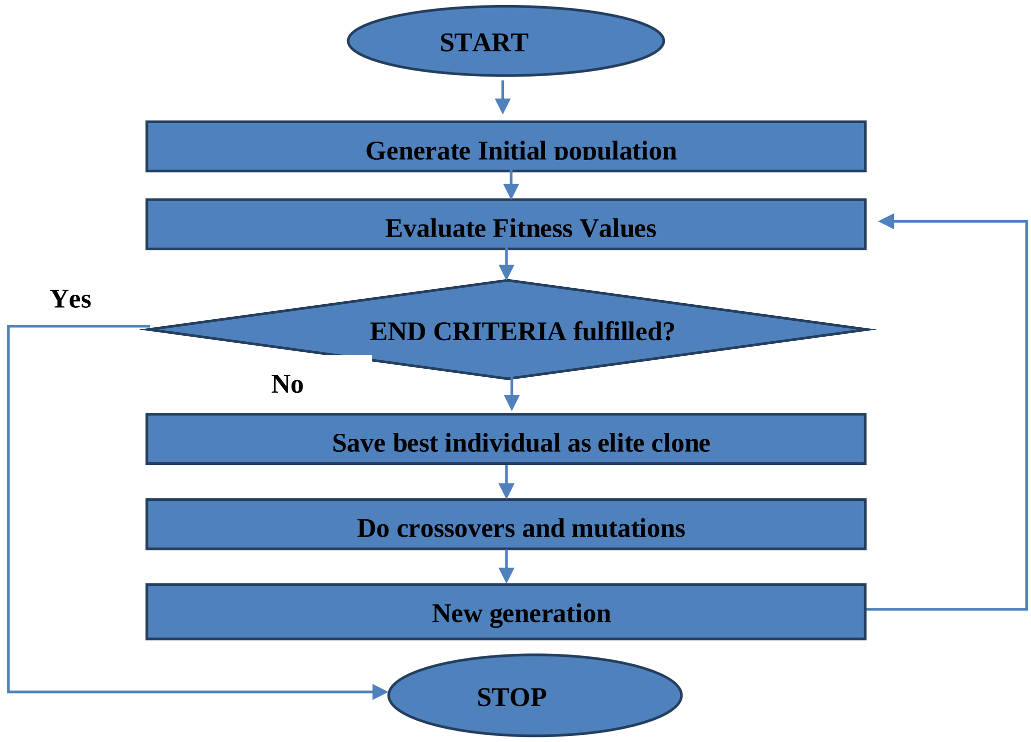

Fig. 1. Flowchart describing the most basic GA.

2.3 Deign considerations

IS 2911(Part 1/Sec 2): 2010 [10] recommends that pile foundations is to be designed in such a way that the load from the structure can be transmitted to the sub-surface with adequate factor of safety against shear failure of sub-surface and without causing such settlement (differential or total), which may result in structural damage and/or functional distress under permanent/transient loading. The pile shaft should have adequate structural capacity to withstand all loads (vertical, axial or otherwise) and moments which are to be transmitted to the subsoil and should be designed according to IS 456: 2000.

2.4 Pile Capacity

In this study the ultimate load capacity of a single pile is obtained by using static analysis, which depends on the reliability of the soil properties for various strata in which the pile rests. The minimum factor of safety on static formula shall be 2.5 as recommended by the code [10]

2.5 Analysis of laterally loaded piles

A pile may be subjected to lateral force for a number of causes, such as, wind, earthquake, water current, earth pressure, effect of moving vehicles or ships, plant and equipment, etc. [10]. The lateral load capacity of a single pile depends not only on the horizontal subgrade modulus of the surrounding soil but also on the structural strength of the pile shaft against bending, consequent upon application of a lateral load. While considering lateral load on piles, effect of other co-existent loads, including the axial load on the pile, should be taken into consideration for checking the structural capacity of the shaft. The IS code [10] suggests that a group of three or more pile connected by a rigid pile cap should be considered to have fixed head condition. In all other conditions the pile should be taken as free headed.

2.6 Structural Capacity

The IS code [10] suggests that the piles should have necessary structural strength to transmit the loads imposed on it, ultimately to the soil. In case of uplift, the structural capacity of the pile, that is, under tension should also be considered.

2.7 Estimation

An estimate of the anticipated cost of work to be carried out is generated by the software. In this study the software generates a design output which includes the structural design of the pile and various specifications involved in its construction. The structural design serves as the necessary drawing for estimation, the other concrete and steel parameters along with workmanship requirements specified by Indian Standards [10] serves as the specifications and the Assam Public Works Department Schedule of Rates [3], as the Rate of Items, while preparing the estimate. All guidelines for estimation provided by Assam Public Works Department has been followed for estimation.

3 Results and Discussions

3.1 Software

The software developed has been designed to optimize pile dimensions based on the column load and the supporting soil parameters. The optimized results can then be used to structurally design the pile foundation or use user specified data for design of the pile. All design procedure has been adopted as per guidelines provided by Indian Standards code [9],[10],[11],[12],[13],[14]. A detailed step by step procedure for using this software for optimization and design of pile foundation has been given below:

1. Under the 'OPTIMIZATION' section the optimization technique is selected as 'GA' from the drop down list provided.
2. Under the 'OPTIMIZATION PARAMETERS FOR PILE' section the Minimum diameter of the pile in mm, Maximum diameter of the pile in mm, Minimum Length of the pile in m, Maximum Length of the pile in m, Biaxial Moment acting in x- direction and y- direction respectively in kNm and the Column Load in kN are entered.
3. Next, the number of soil layers (viz. 1, 2, 3) of stratified soil are selected in the radio buttons provided under 'SOIL PARAMETERS' section.
4. Based on the number of soil layers in stratified soil selected the software highlights only those particular layers for which soil parameter input are needed for computation. In this particular case three numbers of soil layer has been selected .Hence the software opts for soil parameters of three soil layers.
5. Under the Layer section, the Soil type i.e. Cohesive soil or Granular soil for Layers 1 through 3 is selected. Depending upon the type of soil layer selected the software highlights the soil parameters required for that particular soil layer which are required for computation. In this particular case Cohesive soil type has been selected for Layers 1 through 3.
6. The 'Layer length' in m is entered. 'y'= effective unit weight of the soil at pile tip, in kN/m3; '*K*i' = coefficient of earth pressure applicable for the ith layer; 'phi' = angle of wall friction between pile and soil for the ith layer in degrees for Granular soil layer are entered . '*c*p' = average cohesion at pile tip, in kN/m2; 'alpha' = adhesion factor for the ith layer depending on the consistency of soil, 'ci' = average cohesion for the ith layer, in kN/m2 for cohesive soils are entered. In this problem, for layer 3, which is a cohesive soil layer, the software opts the user for cp = average cohesion at pile tip, in kN/m2; alpha = adhesion factor for the 1st, 2nd and 3rd layers of the soil respectively, *c*i = average cohesion for the 1st, 2nd and 3rd layers, in kN/m2 for cohesive soil respectively.
7. The software takes all the input parameters and using the GA perform a number of iterations to obtain the optimized pile dimensions within the limits. The Optimized results are displayed under the 'OPTIMIZED RESULTS OF PILE' section as Length of pile in m and Diameter of pile in mm. In this problem the optimized results are: Length of pile is 14 m and Diameter of pile is 450mm.
8. User may opt to use the optimized pile dimension for structural design by checking the 'Use Optimized Parameters for Design' check box under 'STRUCTURAL DESIGN PARAMETERS FOR REINFORCED CONCRETE PILE' section or provide the Length and diameter of the pile under the same section .In this case optimized parameters has been used for design.
9. The Vertical Load in kN (if optimized parameters not used), Effective eccentricity of vertical load in mm, Lateral Load in kN, Point of Lateral Load application from Ground Level in m and Modulus of subgrade reaction in MN/m3 are entered. The Grade of Steel in N/mm2 (viz. Fe 415, Fe 500), Grade of concrete in N/mm2 (viz. M25, M30), Clear Cover in mm (viz. 50mm , 75mm), Cover to reinforcement in mm (viz. 50mm , 75mm), Diameter of Longitudinal bars in mm (viz. 12mm , 16mm, 20mm) and Diameter of tie bars in mm (viz. 8mm, 10mm) from the respective drop down lists provided under ' STRUCTURAL DESIGN

PARAMETERS FOR REINFORCED CONCRETE PILE' section are selected. The structural parameters used in this problem are represented in Fig. 2 .

10. The software generates the Design results under 'PILE DESIGN RESULT' section with a labelled Diagram of the pile with reinforcement detailing.
11. Under the 'ESTIMATED COST 'section, the Schedule of Rates from the drop down list provided (Here, APWD 2013-14 [3]) is selected. The software generates an estimated cost of the pile in Rupees.

A stand-alone executable file is created with the use of Matlab Compiler in order to allow for users without a Matlab license to install and run the software on different operating systems.

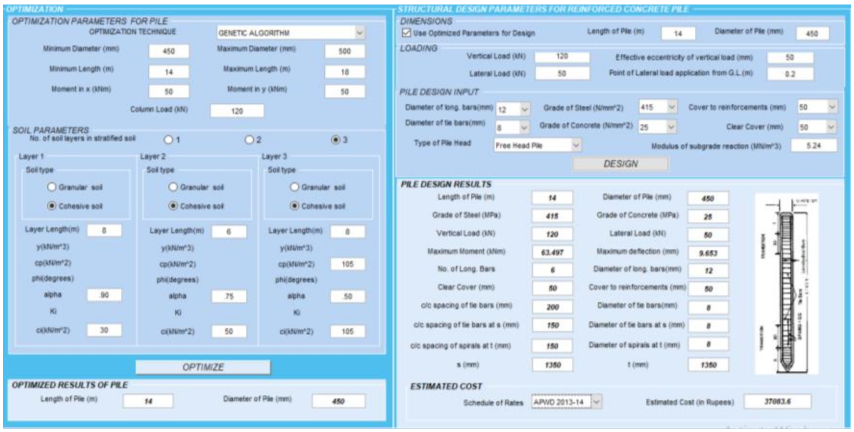

Fig. 2. Pile Foundation Optimization and Design software interface

The entire process can be summarized in the flow diagram (see. Fig.3)

3.2 Calibration of software

A few numerical examples' data has been adopted from a few renowned books [20],[23],[24] in the field of geotechnical engineering for calibrating the developed software. Methodology discussed in the previous section has been adopted for optimizing and designing the pile. This also serves as a manual verification of the results generated by the software.

The basic form of the GA used is as follows:

<u>Optimize:</u>

$$Q_u = A_p\left(\frac{1}{2} D_\gamma N_\gamma + P_D N_q\right) + \sum_{i=0}^{n} K_i P_{Di} \tan\delta_i A_{si} \quad (1)$$

$$\text{or } Q_u = A_p N_C c_p + \sum_{i=0}^{n} \alpha_i c_i A_{si} \quad (2)$$

depending upon the nature of the soil layers. The first term gives the end-bearing resistance (Qp) and the second term gives the skin friction resistance (Qs).

<u>Subject to the constraints:</u>

1. Ultimate Load Capacity of the pile (Qu) ≥ 2.5 x Column Load (Q).

2. Optimized Length of pile (L) in m ≥ Minimum Length of pile (Lmin) specified by user in m and Optimized Length of pile (L) in m ≤ Maximum Length of pile (Lmax) specified by user in m.
3. Optimized Diameter of pile (D) in mm ≥ Minimum Diameter of pile (Dmin) specified by user in mm and Optimized Diameter of pile (D) in mm ≤ Maximum Diameter of pile (Dmax) specified by user in mm.
4. Optimized Length of pile (L) in m>0.
5. Optimized Diameter of pile (D) in mm>0.

The results of software generated and Manual design tallies with those of manual calculations. Hence the working process of the software has been correlated with those of a standard. This shows that the software is capable of generating acceptable optimized and design results

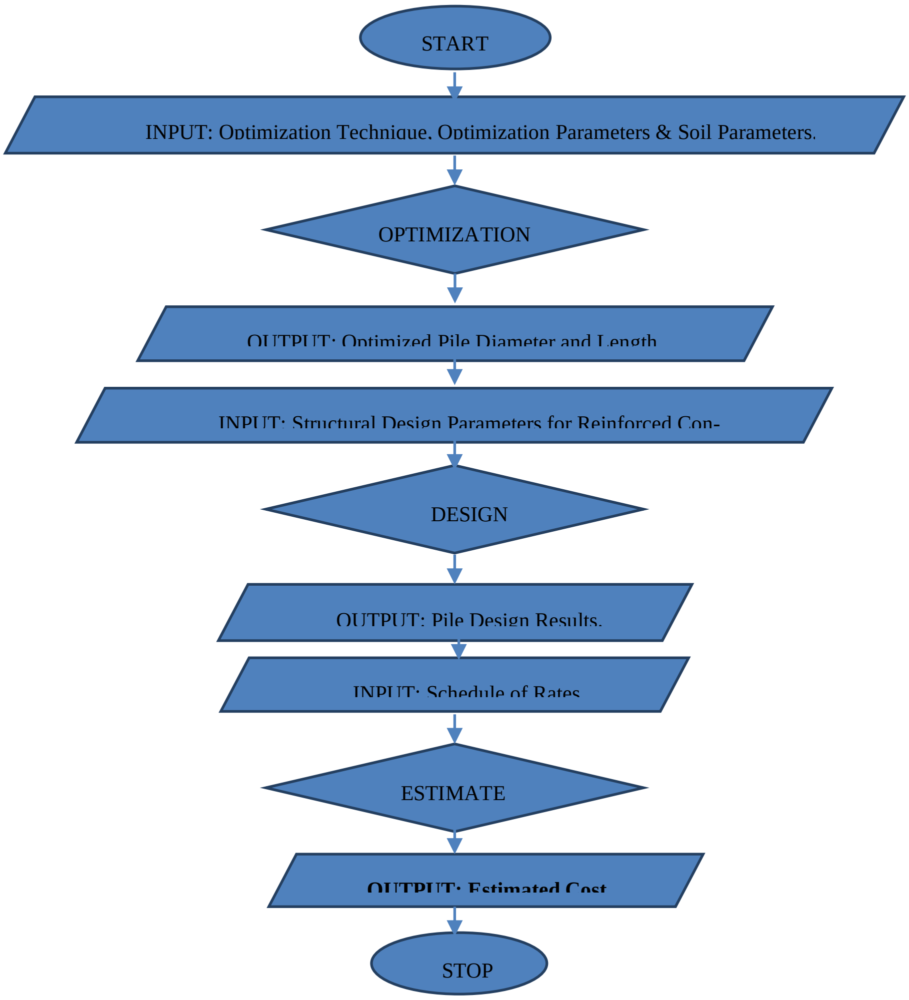

Fig. 3. Flow Diagram of Pile foundation Optimization and Design

3.3 Findings and interpretations

Based on the Results of Optimization and Design obtained from the software and supported by its manual verification, it can be inferred that:

1. The proposed software is capable of using a GA for optimization of Geotechnical Design parameters of a pile for a given set of variables representing the supporting soil parameters.
2. The proposed software is capable of automating Optimization process by incorporating code based and user defined input parameters and limits in Geotechnical Design.
3. Acceptable structural design is achieved by incorporating optimized parameters and/or user defined parameters for design satisfying all Indian Standards [10],[14] codal provisions.
4. The software is capable of generating Structural Design based on the optimized results and/or user defined parameters following all Indian Standards[10],[13],[14],[25],[26] specifications.
5. The software is capable of providing a user friendly interface for providing a real time optimization and design environment for Engineers, providing users partial or full control over the entire Optimization and Design process.
6. The software is also capable of generating preliminary estimates of conceptual design, based on Schedule of Rates [3] and recommended guidelines, for better selection of a particular design.

3.4 Implications

The software developed through this study has adopted the design methodology and constraints considering all the variables involved in pile foundation design and construction recommended by Indian Standards code [9],[10],[11],[12],[13], which is widely used in practice in India. The software also provides provision for a real time optimization and design environment along with provisions for user defined practical considerations or limits in design which may provide ease in practical implementations of the conceptual design. This may cater to shorten the gap between conceptual optimization and design outputs and the actual field applicability. The software also provides an estimate of the design produced so as to allow the designer to better judge the applicability of a particular design

4. Conclusion and Further Scope

The software codes developed has its own limitations. However, based on the asumptions in this study, the code developed using a GA has suitably demonstrated its usefulness in the optimization of the pile design process. This code may later on be developed for being applied in the optimization of piles in group action. This will required further attention to the application of GA in Geotechnical Engineering.

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