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Ultimate Bearing Capacity of Strip Footing on Reinforced Embankment using Upper Bound Limit Analysis

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Abstract. The present investigation determines the ultimate bearing capacity of a surface strip footing resting on a reinforced embankment. The analysis is performed using the upper bound limit analysis, along with a multi-block failure mechanism. In this study, two factors named the increment factor (E_f), and the influence factor (R_f) are introduced to determine the effect of reinforcement and embankment slope on the bearing capacity, respectively. The influence of set-back distance (S_L), slope angle (β), cohesion (c) and angle of internal friction (ϕ) of soil, and reinforcement depth (S_v) on the magnitude of E_f and R_f is explored. Soil is assumed to follow the Mohr-Coulomb failure criterion along with the associated flow rule. While determining the influence of the reinforcement on the bearing capacity, the reinforcement is assumed to be a strong one, i.e. the tensile strength is much higher than the force induced in the reinforcement. It can be conceived that the magnitude of R_f greatly depends on c, ϕ, S_L and S_v.

Keywords: Collapse Mechanism, Embankment, Limit Analysis, Soil Reinforcement, Strip Footing.

1 Introduction

In several occasions, foundations are placed on slopes such as highway or railway resting on embankments. It is understood from the literature [4,5,10,11] that the bearing capacity of a footing resting on the sloping ground is generally found lower than that of a footing resting on the horizontal ground. Nowadays, the application of reinforced embankment has been a fascinating concept among geotechnical engineers. Hence, the bearing capacity of footings resting on such reinforced slopes can be worth exploring [3,9,12,15]. The present investigation determines the ultimate bearing capacity of a surface strip footing resting on a single-layer reinforced embankment, as shown in Fig. 1a. The analysis was performed using the upper bound limit analysis, along with a multi-block failure mechanism. In this study, an influence factor (R_f) is introduced, which can be multiplied with the limit load of the footing resting on the horizontal soil bed to determine the effect of the embankment slope on the bearing capacity. Similarly, an increment factor (E_f) is introduced to determine the effect of reinforcement on the bearing capacity. The influence of setback distance (S_L), slope angle (β), cohesion (c) and angle of internal friction (ϕ) of soil, and embedment depth (S_v) on the magnitude of E_f and R_f is explored (Fig. 1a). The results are provided based on a parametric study so that it can be used to calculate the limit load of a strip footing resting on a reinforced embankment. While determining the influence of the reinforcement on the bearing capacity, the reinforcement is assumed to be a strong one, i.e. the tensile strength is much higher than the force induced in the reinforcement. Hence, the reinforcement is considered to fail due to slippage. Soil is assumed to follow the Mohr-Coulomb failure criterion along with the associated flow rule. It can be conceived that the magnitude of R_f greatly depends on c, ϕ, S_L and S_v.

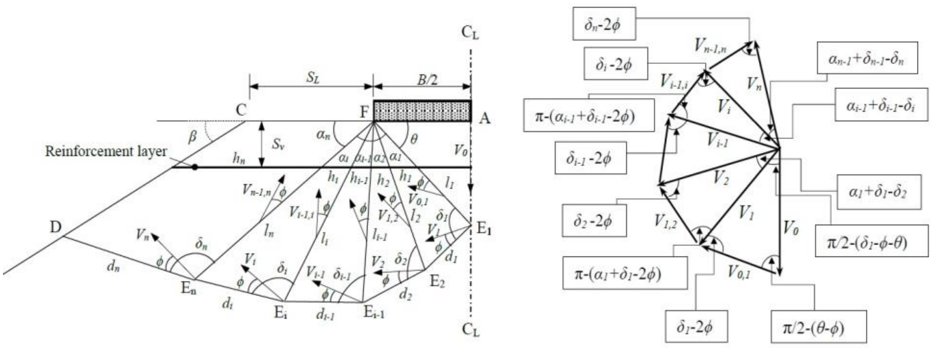

Fig. 1. a) Failure mechanism and velocity vectors, b) Velocity hodograph.

2 Problem Definition

A perfectly rough surface strip foundation of width B rests on the top of a single-layer reinforced soil embankment with a setback distance S_L on either side of the foundation, as shown in Fig. 1a. The reinforcement is placed horizontally with an embedment depth S_v. The objective is to determine the bearing capacity of the foundation using the classical upper bound limit analysis based on a kinematically admissible collapse mechanism, as shown in Fig. 1a. The c-ϕ soil in the embankment is assumed to follow the Mohr-Coulomb failure criterion along with the associated flow rule. It is also assumed that the collapse of the footing occurs prior to the failure of the embankment.

Analysis

2.1 Failure Mechanism

Following the work of Biswas and Ghosh [1,2], a kinematically admissible multi-block failure mechanism is assumed in the present study, as shown in Fig. 1a. Taking the advantage of the symmetricity, the analysis was carried out considering half-domain, as shown in Fig. 1a. In this analysis, n number of rigid blocks are considered on either side of the plane of symmetry (C_L-C_L). The collapse mechanism can be defined by the geometric variables α_i, δ_i, and θ, as shown in Fig. 1a. The triangular trapped wedge AE_1F below the footing base is assumed to move along with the footing at the same velocity (V_0). The vertical movement of this trapped wedge causes a lateral movement of the remaining rigid blocks on the left side of the footing. However, the outermost rigid block (FE_nDC) turns out to be quadrilateral due to the presence of the sloping face on the left side. The absolute velocity of the i^{th} block (FE_iE_n) can be presented as V_i, whereas the relative velocity between the i^{th} block and the $(i$-$1)^{th}$ block can be considered as $V_{i\text{-}1,i}$ and so on. The movement of the rigid blocks with different velocities can be confirmed from the velocity hodograph shown in Fig. 1b. The interfaces among the blocks are considered as the velocity discontinuity lines.

It can be seen from Fig. 1a that the reinforcement cuts the rigid blocks with different lengths. Hence, the rate of internal energy dissipation (D) in case of slippage of the reinforcement can be expressed as [7]

$$D = 2l_e(\sigma_n f_b \tan\phi + f_c c)V_l \tag{1}$$

Where, l_e is the effective length of the reinforcement, σ_n is the normal stress acting on the reinforcement, V_l is the relative velocity between the reinforcement and the soil mass, f_b and f_c are the bond coefficients as recommended by Michalowski [8].

2.2 Ultimate Bearing Capacity

As per the upper bound limit analysis, the ultimate failure load of the strip footing can be determined by equating the rate of the external work done with the rate of the internal energy dissipation. Hence, the limit load (P_{urf}) on the footing can be expressed as a function of different geometrical variables of the failure mechanism such as α_i, δ_i, and θ. The least upper bound solution can be obtained by conducting a rigorous optimization study. Hence, the ultimate bearing capacity of the footing (q_{urf}) can be expressed as

$$q_{urf} = \frac{P_{urf}}{B} = \left[\frac{c\cdot\left(f_3+f_4+f_5+f_c\cdot M_{fc}\right)+0.5\cdot\gamma\cdot B\left(-f_1-f_2+2\cdot\mu\cdot\frac{S_v}{B}\cdot M_{f\gamma}\right)}{\left(1-\mu\cdot\frac{S_v}{B}\cdot M_{fp}\right)} \right] \tag{2}$$

Where,

$$f_1 = 0.5\tan\theta$$

$$f_2 = 4\sum_{i=1}^{n}\frac{A_i}{B^2}\cdot\frac{V_i}{V_0}\cdot\sin\left(\delta_i-\theta-\phi-\sum_{j=1}^{i-1}\alpha_j\right)$$

A_i is the area of the i^{th} rigid block.

$$f_3 = \frac{\cos\phi\cdot\cos(\delta_1-\theta-\phi)}{\cos\theta\cdot\sin(\delta_1-2\phi)}$$

$$f_4 = 2\cos\phi \cdot \sum_{i=1}^{n}\left(\frac{d_i}{B}\cdot\frac{V_i}{V_0}\right)$$

$$f_5 = 2\cos\phi \cdot \sum_{i=2}^{n}\left(\frac{l_i}{B}\cdot\frac{V_{i-1,i}}{V_0}\right)$$

$$M_{fc} = 2\cdot\frac{S_v}{B}\cdot\sum_{i=1}^{n}\left[\frac{h_i}{S_v}\cdot\frac{V_i}{V_0}\cdot\cos\left(\delta_i\text{-}\theta\text{-}\phi\text{-}\sum_{j=1}^{i-1}\alpha_j\right)\right]$$

$$M_{f\gamma} = 2\sum_{i=1}^{n}\left[f_{di}\cdot\frac{h_i}{S_v}\cdot\frac{V_i}{V_0}\cdot\cos\left(\delta_i\text{-}\theta\text{-}\phi\text{-}\sum_{j=1}^{i-1}\alpha_j\right)\right],$$

f_{di} is the ratio of the height of the soil mass above the reinforcement level at the mid-point of the i^{th} reinforcement to the width of the footing

$$M_{fp} = 2\sum_{i=1}^{k}\left[\frac{h_i}{S_v}\cdot\frac{V_i}{V_0}\cdot\cos\left(\delta_i\text{-}\theta\text{-}\phi\text{-}\sum_{j=1}^{i-1}\alpha_j\right)\right] \text{ for } 0\le\left(\theta+\sum_{i=1}^{k}\alpha_i\right)\le 90°$$

$$\mu = f_b \tan\phi$$

3 Results and Discussion

The analysis was performed by writing an inhouse code in MATLAB, and the results are presented in terms of increment factor (E_f) and influence factor (R_f), which can be defined as

$$E_f = \frac{q_{urf}}{q_u} \quad (3)$$

Where, q_u refers to the bearing capacity of a strip footing placed on an embankment without any reinforcement.

$$R_f = \frac{\left|q_{urf}\right|_{\beta>0}}{\left|q_{urf}\right|_{\beta=0}} \quad (4)$$

3.1 Optimum Depth of Reinforcement

The range of the optimum depth of the reinforcement for different values of ϕ was suggested by Michalowski [8] for the horizontal semi-infinite ground. On the contrary, the present study involves a soil embankment with sloping ground surfaces. Hence, the optimal depth for different values of ϕ is calculated by performing a parametric study, and the results are presented in Fig. 2. Accordingly, the depth of the reinforcement layer (S_v) is kept as 0.5B (for ϕ < 40°) and 0.75B (for ϕ = 40°).

3.2 Increment Factor (E_f)

The variation of increment factor (E_f) with various input parameters is given in Table 1. From Table 1, it can be noted that the increment factor for the footing decreases with an increase in the slope angle of the embankment (β). It can also be seen that the increment factor is greatly affected by the variation of c and ϕ. The magnitude of E_f is found to increase with an increase in the value of ϕ, whereas E_f decreases with an increase in c.

3.3 Influence Factor (R_f)

The variation of influence factor (R_f) for different values of c and ϕ with β = 20°, S_L/B =2 is shown in Table 2. It can be observed from Table 2 that the magnitude of R_f decreases as ϕ increases, i.e., the reduction in the bearing capacity becomes higher with an increase in the value of ϕ. However, in case of cohesion, the influence factor does not get much affected by the variation of $c/\gamma B$. From Table 2, it can be noticed that the maximum decrease in R_f is about 25% as ϕ varies from 25° to 35°.

3.4 Collapse Mechanism

The multi-block critical collapse mechanisms generated for different values of ϕ with $c/\gamma B$ = 1, S_L/B = 1, β = 20° and S_v/B = 0.5 are shown in Fig. 3. It can be seen that the extent of the failure zone increases with an increase in the value of ϕ.

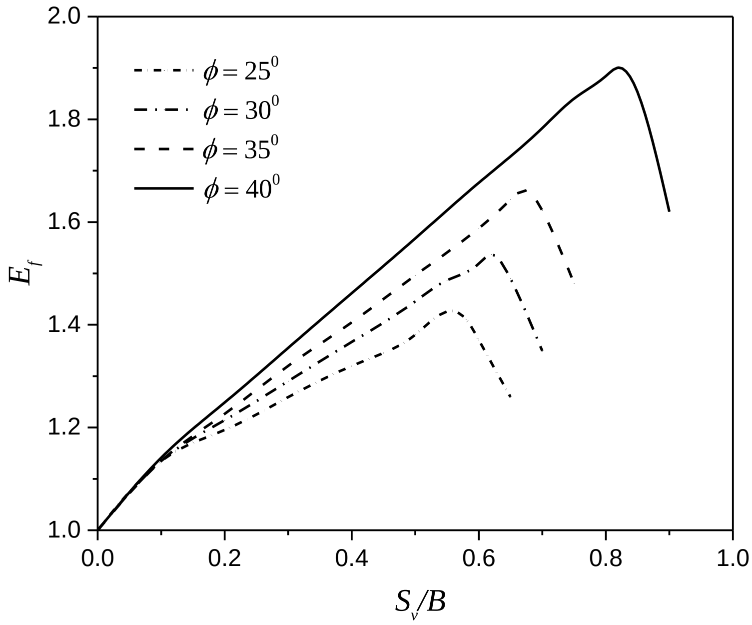

Fig. 2. Optimum depth of reinforcement for different values of ϕ with S_L/B = 1, β = 20°, $c/\gamma B$ = 0.75 for ϕ = 25° and $c/\gamma B$ = 0.5 for ϕ = 30°, 35°, 40°.

Table 1. Variation of E_f with various input parameters.

Φ	$c/\gamma B$	E_f			
		S_L/B = 1		S_L/B = 2	
		β = 10°	β = 20°	β = 10°	β = 20°
25°	0.5	1.47	1.40	1.44	1.41
	1.0	1.46	1.37	1.41	1.38
	2.0	1.43	1.35	1.39	1.37
30°	0.5	1.54	1.45	1.53	1.45
	1.0	1.49	1.41	1.51	1.41
	2.0	1.46	1.38	1.47	1.39
35°	0.5	1.60	1.50	1.60	1.47
	1.0	1.55	1.45	1.55	1.44
	2.0	1.50	1.42	1.51	1.42
40°	0.5	1.98	1.84	1.96	1.79
	1.0	1.88	1.75	1.87	1.73
	2.0	1.79	1.67	1.78	1.66

Table 2. Variation of R_f for different values of ϕ with β = 20°, S_L/B =2

$c/\gamma B$	R_f		
	ϕ = 25°	ϕ = 30°	ϕ = 35°
0.5	0.88	0.77	0.66
1.0	0.87	0.77	0.66
2.0	0.85	0.77	0.67

4 Comparison

Several classical theories [6,14] are available for the determination of the bearing capacity factors (N_c and N_γ) for a strip footing resting on soil without any reinforcement and sloping ground surface. In Table 3, the present values of N_c and N_γ are compared with the upper bound results of Soubra [13]. In Table 4, the present results are compared with the available classical theories [6,8,14]. The values provided by Meyerhof [6] are found to be lower than the present values, whereas the current results find a better match with the results of Vesic [14]. The present results provide a closer match with the upper bound results of Michalowski [8].

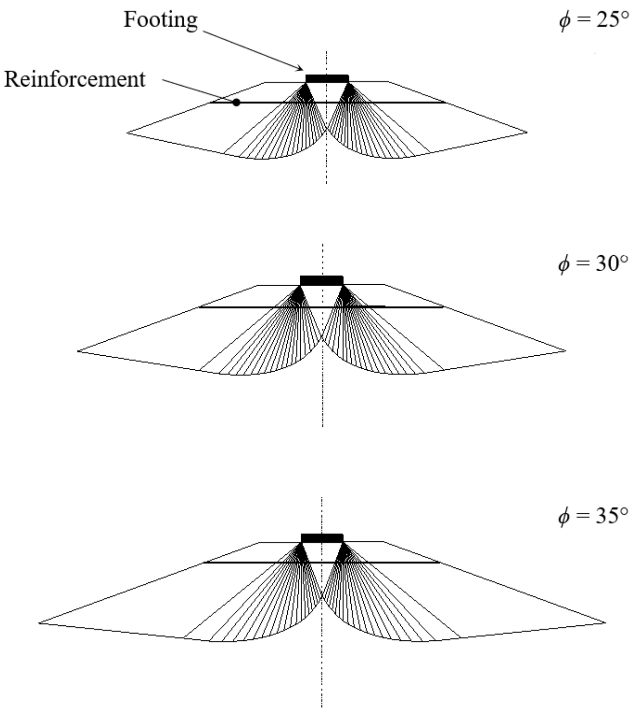

Fig. 3. Collapse mechanisms for different values of ϕ with $c/\gamma B = 1$, $S_L/B = 1$, $\beta = 20°$ and $S_v/B = 0.5$.

Table 3. Comparison of N_c and N_γ values with Soubra [13].

ϕ (°)	N_c		N_γ	
	Present study	Soubra [13]	Present study	Soubra [13]
20	14.84	14.86	4.48	4.49
30	30.15	30.24	21.45	21.51
40	75.36	75.77	119.31	119.84
50	267.20	270.09	1033.04	1042.48

Table 4. Comparison of N_γ values with available literature

ϕ (°)	Present study	Meyerhof [6]	Vesic [14]	Michalowski [8]
20	4.48	2.87	5.39	4.52
25	9.78	6.77	10.88	9.77
30	21.45	15.67	22.40	21.34
35	48.83	37.15	48.03	48.50
40	119.31	93.69	109.40	118.19
45	324.02	262.74	271.80	320.53

The ultimate bearing capacity of an isolated strip footing resting on a reinforced soil bed obtained from the present study is compared with that reported by Michalowski [8]. For $\phi \geq 30°$, Michalowski [8] considered the cohesion of soil and the surcharge as zero and $0.25\gamma B$, respectively. The width of the reinforcement was taken four times the width of the footing. Hence, by utilizing the input parameters adopted by Michalowski [8], the present values of E_f are determined for ϕ = 30° and 40° with the varying depth of the reinforcement layer and compared in Fig. 4. It can be seen that the present values of E_f match reasonably well with those reported by Michalowski [8] for different values of S_v/B.

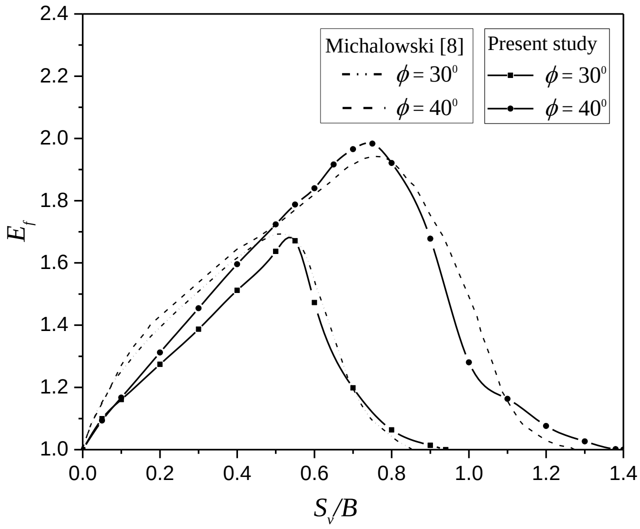

Fig. 4. Comparison of E_f with Michalowski [8] for cohesionless soil

5 Conclusions

The ultimate bearing capacity of an isolated strip foundation resting on a reinforced embankment is investigated using the upper bound limit analysis along with a kinematically admissible multi-block failure mechanism. The results are presented in terms of influence factor (R_f) to represent the effect of the slope on either side of the footing and increment factor (E_f) to capture the effect of the reinforcement. The magnitude of R_f is found to increase with an increase in S_L, but decrease with an increase in ϕ. The value of E_f is found to increase with an increase in ϕ and S_v. In contrast, the cohesion causes a decrement in the value of E_f. The value of E_f is also found to decrease with an increase in β, whereas the setback distance (S_L) does not show any significant effect on E_f. The present results are found to match reasonably well with the results reported in the literature.

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