A Review of Load Transfer Concept of Single Piles considering Soil Non-Linearity

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Abstract: Pile response to a vertical load can conveniently be studied using load deformation behavior of piles. The load deformation behavior of Pile can be determined either by conducting an expensive field test or predict numerically using the in-situ soil parameters. Several approaches have been proposed to predict the load deformation behavior numerically. Two such approaches often used are finite element (FE) analysis and load transfer method. FE analysis is expensive and requires a very skilled person to execute the analysis. The load transfer method is much simpler, easy to use for the analysis and recommended by the API. In the load transfer method, the soil structure interaction has been expressed in terms of t-z curve, the t represents shear stress at the pile shaft, and z is the relative deformation between soil and Pile at the interface. This t-z curve varies with depth, soil-pile interface friction angle, and the pile installation. This paper critically reviews several t-z curves based on the theoretical, empirical, semi- empirical, elastic and non-linear methods that have been used in the past to study the load transfer concept of the single Pile. The development, limitation and range of applicability of these models are also explored in detail.

Keywords: Soil Non-linearity; Pile foundation; load deformation curve; Soil-Pile interface

1.Introduction

Pile Foundations are often used to transfer loads from the superstructure to the sub-structure. Piles foundation is versatile and can be considered a reliable foundation system that can provide support to axial load, lateral load, and moment. The Pile transfers the load to the soil through skin friction and tip resistance. The load deformation response of a pile is an essential part of a pile foundation. Generally, static pile tests are performed in the field to determine the load-deformation behaviour of piles. These tests are very expensive, time-consuming, require a lot of resources and are very rigorous. Hence, the prediction of load deformation of axially loaded piles using routine soil parameters may be considered a suitable way to study the pile foundation. Several prediction approaches have been developed, which can be categorized as simplified theoretical methods (Poulus and Davis (1968),Randolph & Wroth(1978), Song et al (2018)), boundary element methods (Butterfield & Banerjee (1971), Mandolini, A.Viggiani, C. (1997), de-paiva et al(2000), Wong et al(2005), Ai & Han (2009)), load transfer method (Vijayvergiya (1977), Kraft (1981), Lee & Xiao

(2001), Zhu & Chang (2002), Castelli & Maugeri (2002), Wang et al (2012), Nanda & Patra (2014), Zhang et al (2016), Boonyatee et al (2017) , Lu & Luo (2018), Liu et al (2019), Wang et al (2020), Chen et al (2021)) and finite element methods (Trochanis et al 1991), S.Henke (2009), Jun Ju(2015), Mangalthu & Zhao (2017), Salgado & Prezzi (2019)etc.

The Finite Element Method (FEM) and the Boundary Element Method (BEM) are more versatile, robust, capable of implementing various soil models and useful for estimation of pile response under different conditions, but these methods are mainly limited to elastic problems that ignore the elastoplastic soil-pile response. FEM and BEM also need highly skilled persons for the analysis. Both methods involve a lot of Numerical complexity, can give erroneous results and are expensive. While the Load Transfer Method (LTM), on the other hand is a powerful theoretical method as compared to FEM and BEM, which not only requires less computational effort but can also perform the load-deformation analysis considering the non-linear behaviour of soil (Nanda & Patra (2014), Chen et al (2021), etc). The load transfer method can also accommodate pile installation effects, the stress history and the softening behaviour of soil-pile interface (Wang et al (2020).

Seed & Reese (1957) first proposed the Load Transfer Method (LTM). The LTM uses the concept of Load Transfer Curve (τ-z Curve) where "τ" denotes the mobilized skin friction and "z" denotes the depth of the soil strata. The curve relates the interface Shear Strength (t) to the pile soil relative displacement (z). In LTM, the Pile is divided into several small segments (1to n), each with its own τ -z curve. Figure 1 shows the graphical representation of the load transfer concept. A value of Pile end displacement is assumed at first and then the middle of the nth segment is calculated, which then moves toward n-1 segment following a bottom-to-top approach and finally, the load-deformation curve at the Pile head is achieved following an iterative process. LTM is simple, easy to use and also recommended by the American Petroleum Institute. The prediction of load-deformation behavior heavily depends on the accuracy of the τ -z curve.

Recently several methods have been used to determine the τ -z curve, some based on linear analysis (Randolph & Wroth (1978), Salgado & Prezzi(2013),Ai & Han (2009)) and others based on Non-linear Analysis (Kraft et al. (1981),Lee et al.(2001),Zhu & Chang (2002),Wang et al.(2012),Nanda & Patra (2014),Zhang et al.(2016),Chen et al.(2021). The purpose of this paper is to critically review some of the theoretical τ -z models available in the literature.

2.Load Transfer Concept and Pile-Soil Interaction (LTM)

The load transfer behaviour of pile-soil interaction is an important aspect of the design of pile foundations. When a pile is loaded vertically, there will be deformation in Piles. Due to relative deformation, shear stress develops at the pile-soil interference, affecting the soil around it. The load can either be transferred from Pile to soil or from soil to pile based on the relative deformation of soils and piles. The mechanical behaviour of the pile soil interface is complex and involves a lot of uncertainties. Factors such as the pile roughness, pile geometry, composition of the soil, relative density, shear modulus, shape and size of soil particles, state of soil after installation of the pile, end bearing conditions, etc affect the behaviour of the soil pile interaction at the interface. Several studies have been conducted to explore the intricacy of soil-pile interaction and subsequent load carrying capacity of piles (Coyle and Reese (1966), Randolph & Wroth (1978)). The load transfer concept was first introduced by Seed & Reese (1957) where the soil is considered a spring material and the behaviour of these springs (Fig.1) is expressed in terms of load transfer or τ -z curve. These springs resist the vertical displacement of the pile shaft. The advantage and limitation of τ -z curve has been presented in Table 1

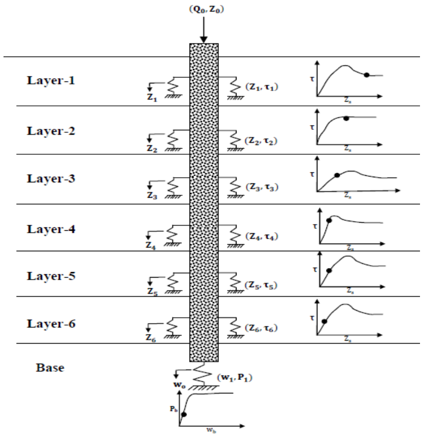

Fig. 1 Load-transfer concept for a vertical compressible pile

When a load of Q_0 is applied at the pile head, it produces deformation along the length of the Pile starting from pile head to Pile tip. The skin resistance along the pile surface and the stress at the base that the deformation causes can both be depicted by the τ-z curve. There are several τ -z curve have been proposed which can be classify in to two category; (1) Elastic (Randolph and Worth, 1979; Coyle and Suleiman, 1967; API for sand, 2002) (2) Nonlinear (Kraft et al., 1981; Lee et al., 2001; Briad and Tucker, 1984; Nanda and Patra, 2014; Mosher 1984; Vijayvergiya, 1977; Zhu and Chang, 2002; Wang et al., 2012; Zhang et al., 2016 and Coyle and Reese, 1966). Based on its formulation, the τ -z curve can further be classified as empirical (Coyle and Reese, 1966; Coyle and Suleman, 1967; Vijayvergiya, 1977; Briad and Tucker, 1984; Mosher, 1984; API, 2002), theoretical (Chen et al., 2021; Randolph and Worth, 1979; Kraft et al., 1981; Nanda and Patra, 2014; Li et al., 2020 and Lee 2001) and semi-empirical (Zhu and Chang, 2002). Some of the important τ -z curve discussed briefly below.

Table 1. Advantage and disadvantage of LTM

Advantage
It is simple and easy to use in the analysis of single as well as the group of piles embedded in multilayer soils
It can be used considering the soil's nonlinear stress-strain behaviour.
The installation effect can easily be included in the analysis by incorporating the properties effected due to installation in the τ -z curve.
Since the API has recommended the LTM therefore the practitioner can use the LTM in real life problem.
Disadvantage

Does not consider the soil continuity.
It is more suitable for drained conditions. Hence it will give long-term load-deformation behaviour of piles.

Coyle and Reese (1966) proposed τ -z curve based on various field and laboratory observations for clay soil. They proposed three set of τ -z curve depending on the depth of Pile. The variation is nonlinear and one of the curve possess softening behaviour. The limitation of this set of τ -z curve is that the curve is fixed and not changing with the soil properties and the field condition.

Coyle and Suleiman (1967) proposed τ-z curve by correlating the laboratory results with data from field tests of instrumented piles in sand. They proposed two sets of τ-z curves based on pile depth. Two curves represent τ-z curve for depth of 0 to 6.1m (0 to 20ft) and beyond 6.1m (20ft) respectively. The τ-z curve is linear in nature and no softening behaviour was considered. It is observed that the ratio between skin friction and soil shear strength is 0.5 and 2 for first two curves.. The limitation is same as Coyle and Reese (1966) with addition that the variation is linear.

Vijayvergiya (1977) proposed a mathematical relationship for a nonlinear t-z curve as follow:

$$\frac{\tau}{\tau_m} = 2\sqrt{\frac{W}{W_c}} - \frac{W}{W_c} \qquad [1]$$

Where, W = displacement of pile shaft at skin friction of τ, W_c = displacement of pile shaft at skin friction of τ_m, τ_m = maximum skin friction

It is assumed that at $W>W_c$ the value of τ is equal to τ_m. Softening behavior is not included in the t-z curve. The limiting values of τ_m are 95.76 kPa, 81.39 kPa, 67 kPa and 47.88 kPa for medium dense soil, silty soil, sandy silt soil and silt soil respectively. The value of W_c ranges from 5.08mm to 7.62mm (0.2 to 0.3in). The limitations in τ-z curve by Vijayvergiya (1977) are given below:

(i) the pile geometry is not considered

(ii) τ_m is almost fixed irrespective of pile depth and soil stiffness has not been considered.

Randolph and Wroth (1978) proposed a theoretical method for t-z curve. The method includes soil stiffness and pile geometry. The expression of τ-z curve is

$$Z_s = \frac{\tau r_o}{G}\ln\left(\frac{r_m}{r_o}\right) \qquad [2]$$

Where, G= shear modulus, τ = Shear stress mobilized at the pile-soil interface, r_o= Pile radius, r_m= is a radius, beyond which strain is supposed to be zero. The average value of r_m along the pile length L proposed by Randolph and Wroth (1978) can be expressed as

$$r_m = 2.5L\rho(1-\mu) \qquad [3]$$

ρ = the ratio of soil shear modulus at mid depth to that of the pile tip; μ = Poisson ratio

Kraft et al. (1981) modified the Randolph and Wroth's equation for t-z curve by introducing the nonlinear soil stress-strain behavior. The expression for t-z curve is

$$Z_s = \frac{\tau r_o}{G_0}\ln\left(\frac{\frac{r_m}{r_o}-\left[\frac{\tau R_f}{\tau_m}\right]}{1-\left[\frac{\tau R_f}{\tau_m}\right]}\right) \qquad [4]$$

R_f = stress-strain curve fitting constant, G_0 = initial shear modulus

The model used for nonlinear soil stress-strain behavior is similar to the model given by Duncan and Chang (1970). Kraft et al. (1981) further extend the τ-z curve for capturing Pile softening behavior. They proposed a method in which softening observed in a direct shear test can be scaled to the field condition. The assumptions are the assumptions given in the model by Randolph and Wroth (1978).

Briaud and Tucker (1984) proposed t-z and p-w curve considering locked-in or residual stress. The residual stress is the locked-in stress developed after driving the piles. Both the curves follow the hyperbolic variation. Briaud and Tucker (1984) further analysed thirty-three (33) pile load test data and proposed empirical relationships for input parameters.
Expression for tip resistance and deformation (p –w curve) is

$$P = \frac{W}{\frac{1}{K_{p1}} + \frac{W}{P_{max} - P_{res}}} + P_{res} \qquad [5]$$

Expression for t-z curve is

$$\tau = \frac{Z_s}{\frac{1}{K_\tau} + \frac{Z_s}{\tau_{max} + \tau_{res}}} - \tau_{res}$$

Where, K_{p1}, K_f = are the initial stiffness, P_{res} = residual load, τ_{res}= residual stress,W= pile tip movement, Z_s= pile shaft movement
Mosher (1984) proposed hyperbolic expression for t- z curve. The expression for t-z curve is

$$\tau = \frac{Z_s}{\frac{1}{K_f} + \frac{Z_s}{\tau_{max}}}$$

K_f = intial stiffness, τ_{max} = ultimate side friction
Using field observation Mosher (1984) proposed K_f value corresponding to angle of internal friction and produced a design chart to determine the ultimate side skin friction corresponding to the ratio between pile depth and diameter of the pile at a given ϕ value. The API (2002) recommended graphical τ-z curves based on a large amount of data base are shown in Figure 4. For sand, the variation of τ-z curve is linear and no softening behaviour is considered. In the case of clay soil, the variation of τ-z curve is nonlinear and softening behaviour is considered. The recommended values of t_{res}/t_m are in the range of 0.7 to 0.9. The limitations are the soil stiffness and nonlinear soil behaviour are not considered and the maximum deformation remains constant along the pile depth.

Zhu and Chang (2002) proposed a formulation for t-z curve. They introduced nonlinear soil stress-strain behaviour in τ-z curve by using the relationship given by Fahey and Carter (1993). This model required pressure meter test result to develop τ-z curve. The expression for t-z curve is

$$Z_s = \frac{\tau r_o}{G_o g} \ln\left[\frac{\left(\frac{r_m}{r_o}\right)^g - f\left(\frac{\tau}{\tau_m}\right)^g}{1 - f\left(\frac{\tau}{\tau_m}\right)^g}\right] \qquad [6]$$

Where, f and g are the fitting parameters that can be obtained from the pressure meter test. Softening behaviour has not been considered in τ-z curve.

Nanda and Patra (2014) proposed an analytical method to develop τ-z curve. This approach includes degree of nonlinearity, hardening and softening behaviour at pile-soil interface., τ-z curve for pre peak and post peak have been proposed using nonlinear soil stress-strain behaviour. The pre peak τ-z curve expressed as

$$z_s = \frac{C}{G_o}\left\{\tau r_o \ln\left(\frac{r_m}{r_o}\right) + \frac{\tau r_o f}{1+f}\left[\left(\frac{\tau}{K\tau_m}\right)^f Y - \left(\frac{\tau r_o}{K\tau_m r_m}\right)^f X\right] + Z(r_o - r_m)\right\} \qquad [7]$$

X = hypergeom $\left\{[1,1,\mathrm{f}+1],\left[2,2+\frac{1}{\mathrm{f}}\right],\left(\frac{\tau \mathrm{r_o}}{\mathrm{K_1}\tau_{\mathrm{m}}\mathrm{r_m}}\right)^{\mathrm{f}}\right\}$, Y=hypergeom $\left\{[1,1,\mathrm{f}+1],\left[2,2+\frac{1}{\mathrm{f}}\right],\left(\frac{\tau}{\mathrm{K_1}\tau_{\mathrm{m}}}\right)^{\mathrm{f}}\right\}$, Z= $\tau_{\mathrm{o}}\left(\mathrm{B}-\frac{1}{\mathrm{C}}\right)$

$$\mathrm{r_m} = 2.5\mathrm{L}\rho(1-\mu)$$

Where, τ = Mobilized Shear stress along the pile-soil interface; r_o= Pile radius; r_m= is a radius, beyond which the strain is assumed to be zero. z_s = Pile shaft movement at shear stress τ, ρ = the ratio of soil shear modulus at mid depth to that of the pile tip; μ = Poisson ratio.

Chen et al. (2021) gave a load transfer model for axially loaded piles installed in modified cam clay (MCC) soils under undrained loading. The rigorous deformation process of the soil surrounding the pile shaft during undrained loads is the basis for the model's Lagragian formulation. When Pile is loaded initially, it behaves elastically and with increase in load the soil at the pile-soil interface yields first and then a plastic zone is formed in the pile vicinity. Hooke's law models the former part whereas the later part which shows the elastoplastic behaviour of soil is represented by MCC model. Elastoplastic displacement at the plastic region and elastic displacement at the elastic region combine to create the total displacement. Therefore, the entire t-z curve of a horizontal soil slice is obtained by combining the two regions -

Elastic Model

$$U_z^e = \int_{r_m}^{r_s} tan\,\gamma_{rz}^e(r)dr = \int_{r_m}^{r_s} \tan\frac{r_s}{r}\frac{\tau_{rz,s}}{G}dr \qquad [8]$$

Elastoplastic Model

$$U_z^p(r) = \int_{r_p}^{r} tan\,\gamma_{rz}(r)dr = \int_{r_p}^{r} tan\,(\gamma_{rz,p} + \int_{\tau_{rz,p}}^{\frac{r_s}{r}\tau_{rz,s}} \frac{T}{T_{44}} D\tau_{rz})dr \qquad [9]$$

Hence, the final equation becomes: -

$$U_z = \begin{cases} \int_{r_m}^{r_s} \tan\frac{r_s}{r}\frac{\tau_{rz,s}}{G}dr, & \tau_{rz,s} \le \tau_{rz}, \\ \int_{r_m}^{r_s} \tan\frac{r_s}{r}\frac{\tau_{rz,s}}{G}dr + \int_{r_p}^{r} tan\left(\gamma_{rz,p} + \int_{\tau_{rz,p}}^{\frac{r_s}{r}\tau_{rz,s}} \frac{T}{T_{44}} D\tau_{rz}\right), & \tau_{rz,s} \ge \tau_{rz} \end{cases} \qquad [10]$$

Many simplifying assumptions were also considered for the study which says

(1) Pile soil interface assumed to be perfectly rough

(2) Soil skeleton assumed incompressible under undrained condition

(3) The pile shaft's lateral strain due to compression is disregarded.

These assumptions will result in overestimating load carrying behaviour of flexible piles while underestimating the displacement of pile in strain softening soils. It is to be noted that the pile installation effect was also not considered in the study and therefore it is more useful for short and medium bored piles. To be applied to long driven piles, it will need further calibration and because of the usage of MCC model, it is unable to predict the softening behaviour after the critical state.

Li et al. (2020) presented an analytical method for predicting the non-linear load-displacement behaviour of displacement piles using the exponential load transfer functions. The study intended to consider the pile installation effects but with the limitation that it could be used only for normally consolidated and lightly over consolidated soils and not for over consolidated soils. Basically, two zones were specified –(i) Plastic zone near the pile-soil interface, (ii) Elastic zone away from the interface. Spherical Cavity expansion method was used to simulate the impact of installation around the pile shaft and tip. Generally, the shear modulus taken in load transfer method is the in-situ shear modulus but here the shear modulus after the pile installation is introduced in the following equation

$$W_{se,z} = -\frac{\tau_{s,z} r_0}{G_{0,z}} \int_{r_m}^{r} \frac{1}{(A - Blnr)r} dr$$

The equation can be modified if the radius of plastic region $r_p < r_m$

$$W_{se,z} = -\frac{\tau_{s,z} r_0}{G_{0,z}} \left[\int_{r_p}^{r} \frac{1}{(A-Blnr)r} dr + \int_{r_m}^{r_p} \frac{1}{r} dr\right] \quad [11]$$

Where, $A = \left(\frac{OCR}{2}\right)^{\Lambda} + \chi \left[\left(\frac{3K_0}{1+2K_0} + \frac{1}{\sqrt{3}} M^* \sqrt{OCR-1}\right) + \left(\frac{OCR}{2}\right)^{\Lambda} \left(\zeta ln\left(\frac{r_p}{r}\right) - \frac{\zeta}{2} \pm \frac{\sqrt{4M^2 - 3\zeta^2}}{6} - 1\right)\right]$

$B = \zeta\, \chi \left(\frac{OCR}{2}\right)^{\Lambda}$

The paper concluded that pile installation effects should be considered as it enhances the strength and stiffness of soil around Pile which eventually improves the load carrying capacity.

Lee (2001) used a discontinuous displacement function to limit the plastic soil behaviour to a small annulus around a loaded pile. Soil outside of the annulus was thought to be elastic. An analogous to Duncan & Chang's(1970) hyperbolic model is utilised to account for soil non-linearity which describes the non-linear displacements caused by shear stresses around the shaft. However, the Drucker prager soil model was considered which is an elastic material.

The following hyperbolic model can be used to approximate load transfer functions that have developed at various depths. The elastic vertical soil displacement part (W_{sz}) is taken from Randolph (1978) and lee introduced the nonlinear local shear displacement part (ΔS_z)

$$W_{sz} = \frac{r_0}{G} ln\left(\frac{r_m}{r_0}\right) \tau_z = c\tau_z \quad [12]$$

$$\tau_z = \frac{\Delta S_z}{a + b\Delta S_z}$$

Where The reciprocal of coefficient ‘a’ = the initial stiffness (Ksi) of the shear stress - relative displacement relationship of the Pile–soil interface and,

the reciprocal of coefficient b = the asymptote of the shear stress – displacement curve at a very large value of relative displacement.

Liu et al (2020) constructed an analytical model based on the load transfer theory and the back analysis methods. The link between the distribution function of axial force and the distribution function along the pile length allows for the solution of the differential equation of pile-soil load transfer.

Boonyatee et al (2017) gave a model by decoupling the settlement of pile segment into inelastic part (Z_s) and elastic part (Z_e). The relationship between the inelastic part (Z_s) and mobilised shear stress (τ) is shown by various models by various researchers, such as by a exponential model (Wang 2012),by a softening model (Zhang 2012) and by Hyperbolic model (Lee 2001).

Sheil (2016) presented a model that is predicated on the idea that the mean effective stress following pile installation is independent of the type of soil and linearly decreases with growing normalised radial distance following consolidation. However, the limitation of the model is that it is incapable of reflecting the actual pile installation effects and still needs further calibration and improvement.

Zhang et al. (2012) followed their previous works and proposed two models for pile shaft -soil behaviour and Pile base-soil behaviour. A hyperbolic non-linear model was provided to simulate the behaviour between skin friction and the pile shaft developed along the pile-soil surface, and a bilinear hardening model was used to simulate the load-settlement response developed at the pile base. The overall settlement at the pile shaft is separated into elastic and plastic displacements in Zhang et al. (2016)'s load transfer technique, which was first proposed by Lee & Xiao (2001).

Table 3 summarises the various τ-z curves. Table 2 includes empirical, theoretical, and semi empirical τ-z curves. It is observed that various τ-z curve has been proposed looking into the various soil condition. Installation effect and softening has been included in many of the τ-z curve. τ-z curves are available for both drained and undrained conditions. τ-z curve can be developed used sophisticated soil model like modified cam clay soil model.

Table. 3 Summary of various τ-z curve

Sl no.	Author	τ-z curve behaviour	Remarks
1	Chen et al (2021)	The soil at the pile-soil interface yields first under an increase in axial load, and the formation of a plastic zone around the pile shaft can be described using the MCC model.	Due to the MCC model's foundation, the load-transfer model was unable to account for softening behaviour after critical state. It does not take into account the effect of installing the pile.
2	Wang et al (2020)	Considering Installation Effect, Randolph's equation is modified	Installation effects is considered. (Shear Modulus after pile installation is considered)
4	Zhang et al. (2018)	Elastic solution and Elasto-plastic solution are given separately.	The only failure considered is shear failure at the treated soil-soil interface where the skin friction is also high.
5	Lu & Luo (2018)	Considered the softening behaviour, hence equation for both softening and hardening behaviour is specified.	Softening Behaviour is included.
6	Song et al. (2018)	A generalization scheme using single parameter n is given.	Only Single Fitting Parameter Used.Softening included
7	Boonyatee et al (2017)	A model is given by decoupling the settlement of pile segment into inelastic part (Z_s) and elastic part (Z_e).	Undrained parameters used hence limited to short term settlement
8	Zhang et al. (2016)	Load transfer method is used and overall settlement at the pile shaft is separated into elastic and plastic displacements.	Interaction between the pile cap and soil not considered.
9	Nanda & Patra (2014)	Gave different equations for the pre-peak and post peak behaviour.	Skin friction softening behaviour is included that is often caused by a reduction in the interface friction angle along the residual shear surface formed at the pile-soil interface.
10	Jiu & Huang (2014)	Modelled by load transfer approach.	Different Formulations for flexible and rigid foundations

11	Wang et al (2012)	Modelled by load transfer method with exponential load-transfer functions (BoxLucas1 Function)	The proposed function considered the soil non-linearity in a deep manner.
12	Castelli & Maugeri (2002)	A hyperbolic load-transfer (t-z) function is used to analyse the nonlinear single pile settling behaviour, simulating nonlinear behaviour for both shaft and base resistance.	recognized the importance of soil stiffness nonlinearity and proposed expression for the stiffness efficiency.
14	Lee & Xiao (2001)	Total Shaft displacement =Local Shear Displacement (τ) +Elastic vertical displacement(W)	Local Shear Displacement (ΔS_Z) was also considered for non-linear analysis whereas Randolph considered only the elastic displacement
16	Kraft (1981)	Modified the Randolph and Wroth's equation for t-z curve by introducing the nonlinear soil stress-strain behaviour.	However, the method and presumptions they utilise for the t-z curve are susceptible to stress and strain. Soil softening has been taken into consideration, while strain at failure has received less attention.
17	Randolph (1978)	Approximate Analytical model is presented using the principal of superposition considering the average behavior down the pile shafts separately from that beneath the level of the pile bases.	Major limitation is the entire behaviour is considered linear.
18	Vijayvergiya (1977)	Load transfer method is used for both clay and sand without considering pile geometry.	Model is very Site specific and softening not included, Pile geometry not considered.
19	O Neil (1977)	The Response of Individual Piles within the group is determined by ignoring interaction effect.	True Soil-pile interaction is not considered directly and the convergence of the iterative procedure has not been demonstrated.
20	Poulus and Davis (1968)	Mindlin's equation extended over the corresponding area of the soil	Mindlin's solution are only applicable for elastic soils.

3.Installation Effects

The t-z curve can be affected by the process of installation of Piles. Generally, in case of Bored Piles, no such installation effect is observed but in case of Driven Piles, installation effect is quite prominent and therefore any such effect needs to be analysed carefully while predicting load deformation behaviour of piles. The installation of driven piles results in highly complex conditions developing at the pile-soil interface which are quite often unrelated to the original undisturbed state of the soil or even to the fully remoulded state. Likewise, the installation effect on sand is minimum as compared to clay. Installation of pile in clay strata completely disturbs the state of stress in the soil. After installation of pile, soil close to the pile experiences consolidation and moves vertically downward. Movement of soil produces skin friction on pile shaft which is in general called as negative skin friction or residual stress and sometimes this phenomenon is represented in terms of drag load. The pore water pressure surrounding the pile can vary widely on loading.

Therefore, any elastic theory-based predictions of the transmission of load that do not account for soil disturbance for many diameters around the pile shaft and beneath the toe are unrealistic when taking into consideration the deformation of a pile under its working load. Irrespective of type of soil strata, every driven pile experience locked-in stress or residual stress. The locked-in stress is the shear stress at soil-pile interaction after compilation of installation. Non-inclusion of this locked-in stress in t-z curve may produce error in predicting load-deformation behaviour. When piles are embedded in consolidating clay the locked-in stress develops due to the relative movement of soil and pile. Generally, this behaviour is neglected while predicting load deformation behaviour of piles which may give erroneous results; hence it is very important to account for any installation effect especially in clay during pile driving.

The concept of Residual stress has been discussed in Briad and Tucker (1984) for the first time and it has been included in the t-z curve analysis. They modified the Randolph and Wroth (1978) equation to determine the initial stiffness of t-z curve as shown in equation

$$k_0 = \frac{G}{r_0}\left(\frac{1}{ln\frac{r_m}{r_0}}\right) \qquad [13]$$

$$k_{or} = k_0\left(1 - \frac{\tau_r}{\tau_m}\right)^2 \qquad [14]$$

Where, $\mathrm{k_0} = \text{initial tangent modulus}$
$\mathrm{k_{or}} = \text{tangent modulus at residual stress}, \tau_\mathrm{r} = \text{Residual stress or locked} -$
in stress

Lam et al (2009) conducted centrifuge model tests to investigate compressive loading capacity of piles when pile is subjected to negative skin friction. At higher axial compressive load, the negative skin friction is completely eliminated. The axial load on the pile head which completely eliminated the negative skin friction is in the range of 1.25 to 1.75 times drag load. Recently, Leung et al (2014) proposed an improved analysis method denoted as the Enhanced Multi-Stage Approach where by the incremental lateral soil movement are progressively applied at increasing depths in multiple stages to mimic the actual pile installation sequence.

4.Discussion and Conclusion

1. The LTM is a confiscated method and the prediction of load-deformation behaviour depends on the accuracy of τ-z curve. Over the years, several τ-z curves have been proposed considering various soil and pile conditions. However, the τ-z curve needs many more improvements, as discussed below.
2. Installation effects are only limited to normally consolidated and lightly over consolidated. Based on that, it is concluded that due to the installation effect, the load carrying capacity of pile increases, but the major problem lies in heavily consolidated soils. When installing piles in heavily consolidated soils, negative pore pressure would be produced, resulting in a reduction in load carrying capability (Coop and Wroth,1989, Morrison 1988).
3. The construction of post failure strain softening load transfer (t-z) curves is essentially empirical and important for any non-linear analysis, however, many studies neglected this part.

4. Around 90 % of research work has been devolved based on the solution given by Randolph & Wroth (1978,1979), but the respective solution is confined to the assumption of linear elastic soil behaviour.
5. The majority of t-z curves do not include soil's non-linear stress strain behaviour, softening behaviour, stress history and residual stress (locked in stress) and the quantification of nonlinearity
6. Quantification of nonlinearity has not been considered in t-z curves and The majority of t-z curves have a number of parameters without proper physical meaning
7. Too many parameters are used; some are very hard to determine and lead to erroneous results.
8. The development of an analytical model that takes into account the impacts of pile installation while projecting the load-displacement behaviour of pile groups is necessary.

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