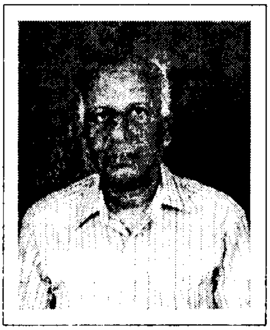
28th IGS Annual Lecture

About the Author



Prof. G. Venkatachalam, Emeritus Fellow in Department of Civil Engineering, Indian Institute of Technology Bombay, joined as a Faculty of I.I.T. Bombay in 1972, is the P.I. of the National Geotechnical Centrifuge Facility since 2003. He graduated in Civil Engineering from BITS Pilani in 1964. He obtained M.Tech degree in Soil Engineering from Indian Institute of Technology Bombay in 1967 and Ph.D. degree from Leningrad Polytechnic Institute 1972 through an Indo-Russian Fellowship Award.

Prof. Venkatachalam has guided 11 Ph.Ds, 60 M.Tech. theses and about 20 projects of B.Tech. students. His fields of specialization are Soil and Rock Mechanics, Remote Sensing and Application to Landslides. He has published 120 technical papers in National/International Journais and Conference proceedings and has coordinated/contributed/edited for Indira Gandhi Open University. He has received several awards from the Indian Geotechnical Society including the Leonard's Prize for best Ph.D. thesis guided in Rock Mechanics.

Prof. Venkatachalam has delivered many invited lectures in India and abroad, held a number of academic and administrative positions including Head of Civil Engineering. He is member of several professional bodies including IGS, ISSMGE, etc. and many expert / technical committees. He has been instrumental in establishing the state-of-the-art geotechnical centrifuge facility at IIT Bombay; has evolved several innovative ideas during the course of research including optimal rotational transformation technique, method for rainfall induced slope instability analysis, method for landslide debris movement modeling, method for fuzzy reliability analysis of slopes, etc. and has been associated in several consultancy projects.

Chairman's Remarks

The Chairman of the Session Maj. Gen. S.N. Mukherjee, a distinguished Geotechnical Engineer and President of Indian Geotechnical Society, introduced the Speaker of the 28h IGS Annual Lecture, Prof. G. Venkatachalam.

He introduced Prof. G. Venkatachalam as an educator and eminent research worker, who has actively participated in several national and international activities to promote geotechnical engineering. Making special mention about the role of field and design practices in geotechnical engineering, he requested Prof. G. Venkatachalam to deliver the IGS Lecture.

Prof. G. Venkatachalam began the lecture with the following remarks :

"Maj. Gen. Mukherjee, President of IGS, distinguished delegates, ladies and gentlemen.

I feel greatly honoured for having been invited to deliver the IGS Annual Lecture. I take this opportunity to thank the Indian Geotechnical Society."

Prof. G. Venkatachalam delivered the 28th Annual Lecture on "Reliability and Risk Analysis of Slopes". The text of the lecture appears as an article in this issue of the Journal.

Vote of Thanks

Dr. K.S. Rao, Honorary Secretary of the Indian Geotechnical Society proposed a vote of thanks.

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Reliability and Risk Analysis of Slopes*

G. Venkatachalam†

Introduction

Geotechnical engineers, the world over, have to frequently contend with limited in-situ investigations and inadequate data. In addition, there are errors and uncertainties associated with the data, which further compound the decision making process. Uncertainties, therefore, are not new to geotechnical engineers, but application of reliability theory to handle them still is.

Conventionally, slope stability is assessed using the deterministic factor of safety. However, a simple computation would show that the deterministic factor of safety could swing to the unsafe side due even to a small variability in the assumed soil properties. It is often not appreciated that, the same factor of safety may connote different levels of slope performance under different conditions and time and that, the changes that would come about due to unforeseeable causes with time, cannot be easily quantified.

Due consideration must be given for the uncertainties arising out of these variabilities. And that is what reliability analysis does. It may be said that, reliability analysis is all about the confidence that can be reposed on the deterministic factor of safety and the two are to be looked upon and used as complementary techniques.

Uncertainties

In recent years, the inadequacies of deterministic analysis have been brought to focus (Duncan, 2000). Uncertainties in geotechnical engineering

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* IGS Annual Lecture delivered at IGC-2005, Ahmedabad on 17 December 2005.

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arise at the exploration stage itself and propagate right up to the performance stage owing to a number of subjective factors as listed below:

1) Exploration Stage

a) field description of subsurface strata in borehole logs leaves scope for interpretation, since it is not always done by a domain specialist

b) descriptions of strata even at the same location could vary when exploration is done by different agencies

c) delineating the boundaries between strata involve subjective averaging

2) Sampling and Testing Stage

a) uncertainties due to spatial variability in properties

b) uncertainties due to statistically inadequate sampling

c) errors due to testing techniques, testing equipment and test conditions not simulating field conditions - (i) systematic errors and (ii) random errors

d) errors due to skill of the testing personnel

e) errors due to scale of the problem – local or regional; some errors of a spatial nature get averaged out when the region is large

3) Analysis Stage

a) level of abstraction / idealization of problem - in this case, Limit Equilibrium Method and its attendant assumptions and idealizations

b) accuracy of computational models used / performance function used

c) uncertainties in estimating triggering factors

d) modelling support measures, e.g. reinforcement

4) Design Stage

a) simplified design philosophies

b) subjective judgement exercised during choice of design parameters

5) Performance Stage

a) intermittent slope modification

b) use of same factor of safety for short and long term performance

Need for Methods to Model Epistemic Uncertainties

Inadequate appreciation of the uncertainties listed above has been a constraint in understanding the usefulness of reliability analysis. It is not uncommon to ignore the uncertainties and adopt a 'safe' design considering the 'worst' combination of applicable parameters. But, engineering challenge lies in understanding them and accounting for them through reliability analysis.

There are several perceptions regarding uncertainties. An excellent idea about uncertainties can be had from Juang et al. (1998), Malkawi et al. (2000), Chowdhury and Flentje (2003) and Christian (2004). Very broadly, they are classified as Aleatory and Epistemic (Baecher and Christian, 2003). The former refers to those due to random variabilities such as spatial variability of soil properties and the latter to those arising out of lack of knowledge. Honjo (2004) points out that geotechnical engineering concerns itself more with the epistemic uncertainties unlike structural engineering, where aleatory uncertainties are given prominence. Aleatory uncertainties are easily modelled by probabilistic methods. But, the epistemic do not have any statistical properties and are amenable to modelling only by fuzzy set theory. An important outcome of this is that, the factor of safety itself is a variable, which is dependant on random and fuzzy soil parameters.

This Presentation

While probabilistic methods to consider the effects of randomness in parameters are well developed, few methods are available for handling fuzziness. Therefore, this paper presents concepts and methods of accounting for fuzzy uncertainties, in particular. Broadly, this paper is divided into four segments:

(i) new point-estimate methods to account for
  a) fuzzy variables
  b) fuzzy-random variables
  c) combination of fuzzy and random variables
  d) system reliability
(ii) GIS-based evaluation of spatial variation of probability of failure
(iii) minimum and system reliability
(iv) GIS-based probabilistic regional hazard evaluation

Integration of methods based on probabilistic principles with GIS and remote sensing based methods is suggested for analyzing spatial problems of slope stability. Validation is attempted through the use of data on failed slopes. Application to reinforced earth walls is illustrated. The usefulness of centrifuge modelling for validation is also pointed out.

Reliability Evaluation

Within the probabilistic framework, the factor of safety is regarded as a random variable. The factor of safety expression is taken as the performance function given by

$$FS = f(c, \phi, \gamma, u, H, \alpha, \ldots) \tag{1}$$

in which
c = cohesion,
ϕ = angle of internal friction,
γ = unit weight,
u = pore water pressure,
H = slope height, and
α = inclination of slope.

Failure of a slope is defined as

$$F = [FS \leq 1] \tag{2}$$

If there are N possible failure surfaces, each with probability of occurrence $P[S_i]$, i = 1, 2, ..., N, then, from the total probability theorem, the probability of failure p_f of the slope is equal to

$$p_f = P[F|S_c]P[S_c] + \sum_{i=1}^{N-1} P[F|S_i]P[S_i] \tag{3}$$

where S_c corresponds to the critical failure surface with $FS = FS_{min}$ and probability of its occurrence $P[S_c]$.

It is also expressed in terms of the difference between the two random variables C and D (capacity and demand functions), i.e. the safety margin S, expressed as

$$S = C - D \tag{4}$$

The safety margin is itself a random variable and hence the probability of failure p_f is the probability that the safety margin will be less than zero.

Mathematically,

$$p_f = P[C < D] \tag{5}$$

Reliability is the probability of successful performance; thus it is the converse of the term probability of failure. For practical structures and performance criteria, it is difficult to compute the probability of failure precisely. Therefore, a first-order estimate is frequently used in probabilistic design, which employs a measure known as the reliability index or safety index β. The number of standard deviations that the mean value of the safety margin ($\overline{S}$) is beyond $S = 0$ is called the reliability index β:

$$\beta = \frac{\overline{S}}{\sigma[S]} \tag{6}$$

A number of approaches have been proposed to calculate β, including the invariant solution by Hasofer and Lind (1974) and the simpler First Order Second Moment (FOSM) method. Taking the performance function and limit state as $FS - 1 = 0$, the reliability index based on the FOSM method is

$$\beta = \frac{E[FS]-1}{\sigma[FS]} \tag{7}$$

where $E[FS]$ is the expected value of the FS and $\sigma[FS]$ is the standard deviation of the FS. The Bishop's simplified method could be used as a basis for evaluating the above two statistical moments of the safety factor in FOSM method and in the Rosenblueth's Point Estimate Method (RPEM). The reliability index can also be expressed in terms of the coefficient of variation of FS as

$$\beta = \frac{E[FS]-1}{E[FS]COV(FS)} \tag{8}$$

If FS is normally distributed, the probability of failure p_f and reliability index β are related by

$$p_f = 1.0 - \Phi(\beta) = \Phi(-\beta) \tag{9}$$

where $\Phi(\beta)$ is the cumulative distribution function (CDF) of the normal distribution.

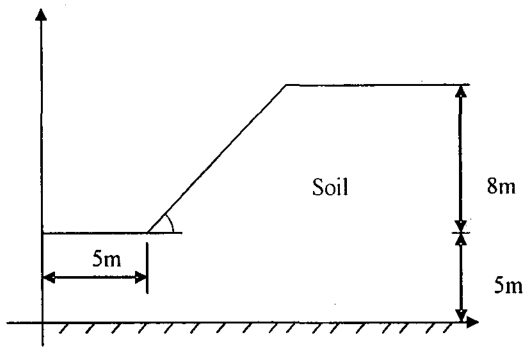


FIGURE 1 : Slope Section for the Illustrative Example

An Evaluation of Probabilistic Methods (FOSM vs. RPEM)

Many studies have been carried out to understand the relative performance of the two methods (Malkawi et al., 2000; Bhattacharya et al., 2004). Consider for example the homogeneous slope shown in Fig.1, with properties given in Table 1.

Central factor of safety of the slope using mean values of the uncertain parameters is 1.243. Let us apply the probabilistic methods.

The Probabilistic Methods

Essentially, the approach involves the estimation of expected value and variance of the random function using statistical moments. Here, two of the most widely used methods for this estimation are briefly explained.

First Order Second Moment (FOSM) Method

The First Order Second Moment (FOSM) method estimates the uncertainty in the factor of safety as a function of the variances of the

TABLE 1 : Soil Data for the Illustrative Example

Soil Parameter	Mean Value	COV (%)
Cohesion, c' (kN/m^2)	18.00	22.22
Friction angle, ϕ' (degrees)	30.00	10.00
Unit weight, γ (kN/m^3)	19.50	3.00
Pore pressure ratio, r_u	0.35	50.00

random input variables, such as angle of internal friction, cohesion, unit weight and pore pressure. It uses Taylor series expansion to estimate the local uncertainty of the factor of safety.

If Y is a function of several random variables, i.e.

$$Y = g(X_1, X_2, \ldots, X_n) \tag{10}$$

one can obtain the mean and variance of Y, using Taylor series expansion, as follows:

$$E[Y] \approx g(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2 g}{\partial X_i \partial X_j} CV[X_i, X_j] \tag{11}$$

where, the derivatives are evaluated at $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n$, and

$$V[Y] \approx \sum_{i=1}^{n}\left(\frac{\partial g}{\partial X_i}\right)^2 V[X_i] + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left(\frac{\partial g}{\partial X_i}\right)\left(\frac{\partial g}{\partial X_j}\right) CV[X_i, X_j] \tag{12}$$

If X_i and X_j are uncorrelated,

$$E[Y] \approx g(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n) + \frac{1}{2}\sum_{i=1}^{n}\frac{\partial^2 g}{\partial X_i^2} V[X_i] \tag{13}$$

and

$$V[Y] \approx \sum_{i=1}^{n}\left(\frac{\partial g}{\partial X_i}\right)^2 V[X_i] \tag{14}$$

Rosenblueth's Point Estimate Method (RPEM)

Another method, known as the Point Estimate Method (PEM), suggested by Rosenblueth (1975) and modified by Li (1992) is widely used. The versatility of the RPEM is that, it can be used even when the functional relationships are not given as an explicit equation. This independence from the type of distribution or correlations among the basic variables is an advantage.

In RPEM, the original probability density function (PDF) of the random variable X is approximated by assuming that the entire probability mass of

X is concentrated at two points x_- and x_+. The calculations are made at two points and Rosenblueth uses the following notation

$$E\left[Y^r\right] \approx p_+ g^r\left(x_+\right) + p_- g^r\left(x_-\right) \tag{15}$$

The locations of x_- and x_+ and the corresponding probability masses p_- and p_+ (so called weighting factors) are:

$$x_- = \mu - z_- \sigma \text{ and } x_+ = \mu - z_+ \sigma \tag{16}$$

$$p_+ = \frac{z_-}{z_+ + z_-} \text{ and } p_- = 1 - p_+ \tag{17}$$

where $$z_+ = \frac{\beta(1)}{2} + \sqrt{1 + \left\{\frac{\beta(1)}{2}\right\}^2} \quad \text{and}$$

$$z_- = z_+ - \beta(1)$$

with $\beta(1)$ being the skewness coefficient of the random variable X. When the distribution of the random variable X is symmetric [$\beta(1) = 0$], $z_- = z_+ = 1$ and $p_- = p_+ = 0.5$, the two points are located at one standard deviation to either side of the mean.

For problems involving N random variables, the r^{th} moment of $Y = g(X) = g\left(X_1, X_2, \ldots, X_N\right)$ about the origin can be approximated as

$$E\left[Y^r\right] = \sum p_{(\delta_1, \delta_2, \ldots, \delta_N)} y^r_{(\delta_1, \delta_2, \ldots, \delta_N)} \tag{18}$$

in which the subscript δ_i is a sign indicator that can only be + or − for representing the random variable X_i having the value of $x_{i+} = \mu_i + \sigma_i$ or $x_{i-} = \mu_i - \sigma_i$, respectively; $p_{(\delta_1, \delta_2, \ldots, \delta_N)}$ is determined by

$$p_{(\delta_1, \delta_2, \ldots, \delta_N)} = \prod_{i=1}^{N} p_{i,\delta_i} + \sum_{i=1}^{N-1}\left(\sum_{j=i+1}^{N} (\delta_i)(\delta_j)\frac{\rho_{ij}}{2^N}\right) \tag{19}$$

where ρ_{ij} is the correlation coefficient between random variables X_i and X_j.

The results of reliability analysis by RPEM and FOSM are compared below in Table 2. For a homogeneous slope the difference in the computed parameters is not significant. But, of the two methods, RPEM is preferable

TABLE 2 : Results of Reliability Analysis by RPEM and FOSM

Values of	Correlation between Variables, c' and ϕ'					
	$\rho(c',\phi') = 0.0$		$\rho(c',\phi') = -0.50$		$\rho(c',\phi') = 0.25$	
	RPEM	FOSM	RPEM	FOSM	RPEM	FOSM
E [FS]	1.242	1.243	1.238	1.243	1.243	1.243
σ [FS]	0.303	0.291	0.282	0.275	0.315	0.3
Overall Reliability Index (β)	0.798	0.835	0.843	0.884	0.771	0.811
p_f (FS as a normal variate) FS $\leq$ 1.0	0 .2124	0.2018	0.1996	0.1883	0.2203	0.2086

because of the ease of computations. Similar observations are made by Bhattacharya et al. (2004) as well.

Therefore, new methods are suggested here for handling fuzzy variables using the point - estimate approach.

Suggested Methods for Fuzzy Uncertainties

The Fuzzy Set Approach

Many studies have been carried out within a probabilistic framework by considering uncertainties to be random (Vanmarcke, 1977; Chowdhury, 1984; Li and Lumb, 1987; Hassan and Wolff, 1999). But, vagueness and imprecision creep in at every stage of a geotechnical activity as outlined earlier. Fuzzy set theory has been developed specially to deal with such cognitive uncertainties that are not statistical in nature (Cremona and Gao, 1997). A convenient way to handle fuzziness is to use an extension of the probability theory (Zadeh, 1968; Dodagoudar and Venkatachalam, 2000a).

In ordinary probability theory, a random variable X in one dimension, is

$$P\{X = x\} = f(x) \tag{20}$$

where $f(x)$ is the probability density of the random variable X, with the condition that,

$$\lim_{x\to\infty} P(x) \equiv \int_{-\infty}^{\infty} dp(x) = 1$$

where, $dp(x) = f(x)d(x)$ and the distribution function is normalizable.

Fuzziness is expressed by a degree of belief in any value of a fuzzy parameter as a membership function or by its so-called α-cuts. Based on this concept, the membership function of a fuzzy parameter can be treated as a weight function with α as its frequency. Now, for a fuzzy-random variable X_A, by analogy with the above equation, we write

$$P\{X_A = x\} \equiv \alpha_A(x) f(x) \tag{21}$$

thereby associating with each x a grade of membership $\alpha_A(x)$ in the set A. Then we define a quantity P(A ; x) as

$$P(A; x) \equiv P\{\alpha_A \leq x\} = \int_{-\infty}^{x} \alpha_A(x') dp(x') = P(A) \tag{22}$$

Corresponding to the normalization condition of ordinary probability theory, we write that

$$\lim_{x \to \infty} P(A, x) \equiv P(A) \tag{23}$$

In practice, since a statistically significant database is seldom available, the use of a subset of a fuzzy set, called fuzzy number suffices (Juang and Elton, 1996). Generally, Triangular fuzzy numbers (TFNs) and Trapezoidal fuzzy numbers (TrFNs) are used to represent the fuzzy input soil parameters to compute the function using Eqn.22.

A simple new approach is presented for considering fuzzy uncertainty (Dodagoudar and Venkatachalam, 2000c). The method is an adaptation of the Rosenblueth's Point Estimate Method and is called here as Fuzzy Point Estimate Method (FPEM). FS obtained by a deterministic method is used as the performance function.

Suggested Point Estimates for Triangular Fuzzy Numbers

A TFN is defined by three values: a minimum, a; a maximum, c; and a mode, b (Fig.2). The mode has the highest membership grade (100%). The values of the mode, minimum and maximum of the TFN are

b = mode = $E[X]$ = expected value of the uncertain parameter,

a = $E[X] - k\sigma[X]$ = minimum value,

c = $E[X] + k\sigma[X]$ E[X] = maximum value, and

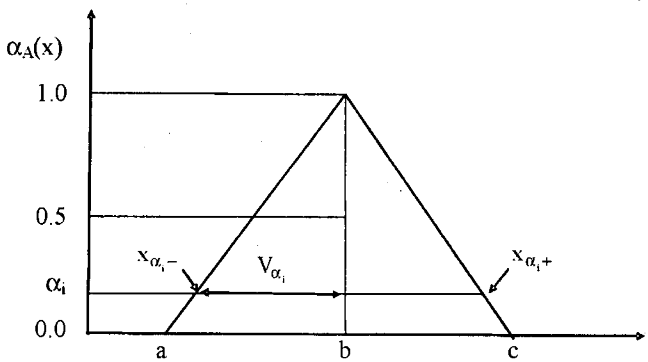


FIGURE 2 : Triangular Fuzzy Number

k = the number of sigma units which will take values 1, 2 and 3 depending on the data available and accuracy of the results desired.

The suggested method makes use of an interval having two points for a particular α-level and reducing the computation of Eqn.22 to a series of interval analyses as follows:

$$x_{\alpha_i \pm} = b \pm V_{\alpha_i} \tag{24}$$

where b = mode of the fuzzy number under consideration,

V_{α_i} = value to be added or deducted to obtain the corresponding parameter point considering its membership value,

$x_{\alpha_i \pm}$ = vector of coordinates of the M uncertain parameters, and

α_i = value of α-level (membership value) chosen for getting intervals for the uncertain parameters.

If there are N α-levels, for a function W = g(X), the sum of the function values at each of the α-level for the case of independent fuzzy variables is

$$w^r_{\alpha_i} = g^r\left(x_{\alpha_i +}\right) + g^r\left(x_{\alpha_i -}\right) \qquad i = 1, 2, \ldots, N;\ r = 1, 2. \tag{25}$$

Now the r^{th} moment of the function is calculated according to

$$E\left[W^r\right]=\frac{\sum_{i=1}^{N}\alpha_i \cdot w^r_{\alpha_i}}{N} \tag{26}$$

Knowing these values one can calculate the probability of failure and reliability index after assuming a suitable distribution for the variable function.

Suggested Point Estimates for Trapezoidal Fuzzy Numbers

Now consider the trapezoidal fuzzy number (TrFNs) shown in Fig.3.

The values of the parameters of the TrFN are:

$$\mathbf{a}=E[X]-k_1\sigma[X]$$
$$\mathbf{b}=E[X]-k_2\sigma[X]$$
$$\mathbf{c}=E[X]+k_2\sigma[X]$$
$$\mathbf{d}=E[X]+k_1\sigma[X]$$

where k_1 and k_2 are the number of sigma units which will take values from 0.5 to 3 depending on the data available and accuracy of the results desired. The points for function evaluation in the parameter space are obtained as

$$x_{\alpha_i+}=c+V_{\alpha_i} \tag{27}$$

$$x_{\alpha_i-}=b-V_{\alpha_i} \tag{28}$$

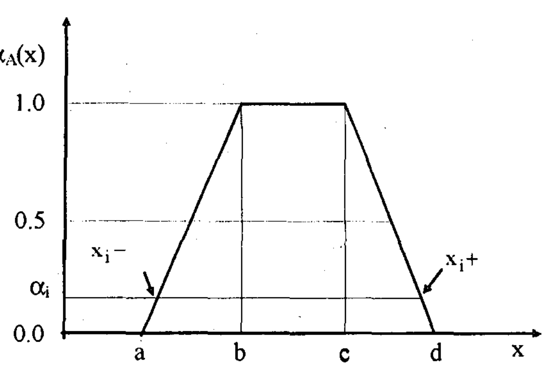


FIGURE 3 : Trapezoidal Fuzzy Number

where c and b = parameter values of the fuzzy number under consideration at $\alpha = 1.0$,

V_{α_i} = value to be added or subtracted to obtain the parameter point

x_{α_i+} and x_{α_i-} = vector of coordinates of the M uncertain parameters, and

α_i = α-level value chosen for getting intervals for the uncertain parameters.

For the performance function W = g(X), the function evaluation can be carried out as below. The sum of the function values at each of the α-level considering the correlation effect between fuzzy variables is

$$w_{\alpha_i}^r = p_+ g^r\left(x_{\alpha_i+}\right) + p_- g^r\left(x_{\alpha_i-}\right) \quad i = 1, 2, \ldots, N; \; r = 1, 2 \tag{29}$$

where p_+ and p_- are weighting factors, given as a first approximation, by the following expression (Dodagoudar and Venkatachalam, 2000b):

$$p_\pm = 1 \pm \sum_{i=1}^{M-1}\left[\sum_{j=i+1}^{M} \frac{\rho_{ij}}{\sqrt{\prod_{i=1}^{M}\left\{1+\left(\frac{\beta(1)_i}{2}\right)^2\right\}}}\right] \tag{30}$$

where ρ_{ij} is the correlation coefficient between fuzzy variables X_i and X_j, and $\beta(1)_i$ is the skewness coefficient of the fuzzy variable X_i. Here, the influence of correlation coefficient is to alter the weighting factors. Now the r^{th} moment of the function is calculated according to Eqn.26. In this approach the maximum α-level used is 0.9.

Since, neglecting higher α-levels implies that the highly possible events are missed, an improvement has been implemented by Mathada et al. (2005) in the form of a possibilistic approach. Accordingly, Eqn.29 is modified to

$$w_{\alpha_i}^r = \frac{p_+ g^r\left(x_{\alpha_i+}\right) + p_- g^r\left(x_{\alpha_i-}\right)}{2} \qquad i = 1, 2, \ldots, N \tag{31}$$

The accuracy increases and the results converge with more number of α-cuts in the proposed approach. The r^{th} moment and the standard deviation of the function are obtained by using the relations:

$$E\left[W^r\right]=\frac{1}{N}\left[W^r_{\alpha_N}+2\sum_{i=1}^{N-1}\alpha_i W^r_{\alpha_i}\right] \qquad r = 1, 2 \tag{32}$$

$$\alpha[W]=\sqrt{E\left[W^2\right]-(E[W])^2} \tag{33}$$

Assuming a suitable distribution function for the FS, the probability of failure and reliability index are evaluated.

Suggested Method for Fuzzy Random Uncertainties

The suggested method is called Fuzzy-Random Probabilistic Method (FRPM). It is applicable to fuzzy variables as well as to random variables with fuzzy means. Let FS be a function of three fuzzy variables (say, c', ϕ' and r_u) and other single-valued non-fuzzy variables. First, fuzzy numbers are constructed for each of the input uncertain variables based on their statistics and the amount of variability to be represented using, say, triangular membership function (Fig.4).

Figure 4 shows a typical membership function with an interval associated with a specific value of α. say I_α. Each of the input uncertain variables can be represented by an interval ($I_{i\alpha}$), at a specific α-cut, where,

$$I_{i\alpha} = \left[m_i, n_i\right] \qquad i = 1, 2, \ldots, N \tag{34}$$

The values of the parameters a, b and c for each of the fuzzy variables are obtained. The desired number of α-cut values is selected. At each α-level, interval values are obtained according to the vertex method. We will get 2^3 points for the function evaluation for three variables. These interval

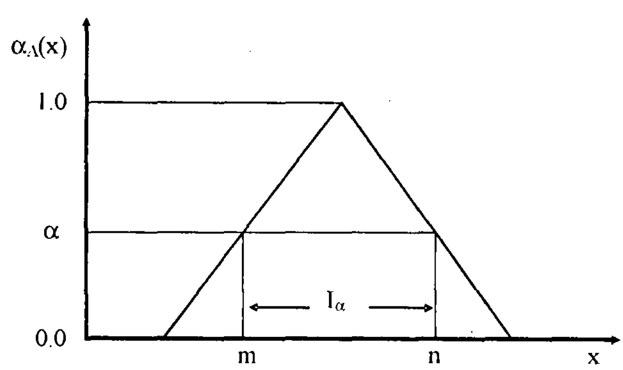


FIGURE 4 : Interval Corresponding to a a-Cut Level on Fuzzy Set A

values of FS constitute the resulting FS fuzzy set. Based on these estimated values of FS at each of the selected α-levels, the E[FS] is approximated by the following expression:

$$E[FS] = w_1 FS_1 + w_2 FS_2 + \ldots + w_8 FS_8 \tag{35}$$

where w_1, w_2, ..., w_8 are the weightages corresponding to each FS values and FS_1, FS_2, ..., FS_8 are the FS values corresponding to each combination of the minimum and maximum values. The standard deviation σ[FS] of FS is approximated by the Taylor series approximation of the function FS retaining only the linear terms. For the case of three uncertain variables c′, f′ and r_u, the σ[FS] is given by

$$\sigma[FS] = \sqrt{\left(\frac{\partial FS}{\partial c'}\right)^2 V[c'] + \left(\frac{\partial FS}{\partial \phi'}\right)^2 V[\phi'] + \left(\frac{\partial FS}{\partial r_u}\right)^2 V[r_u]} + \sqrt{\begin{array}{c} 2\left(\frac{\partial FS}{\partial c'}\right)\left(\frac{\partial FS}{\partial \phi'}\right) CV[c', \phi'] + 2\left(\frac{\partial FS}{\partial \phi'}\right)\left(\frac{\partial FS}{\partial r_u}\right) CV[\phi', r_u] \\ +2\left(\frac{\partial FS}{\partial r_u}\right)\left(\frac{\partial FS}{\partial c'}\right) CV[r_u, c'] \end{array}} \tag{36}$$

A Hybrid Approach for Combined Fuzzy and Random Uncertainties

In geotechnical engineering, more often than not, both random and fuzzy uncertainties would be present simultaneously and there is a need for a suitable approach to handle them. Such approaches are in the development stage. Here, a hybrid method (Venkatachalam et al., 2004) is proposed for the evaluation of safety of slopes in terms of the reliability index, which considers some uncertain parameters affecting safety as probabilistic and others as fuzzy. The result is that the reliability index itself is fuzzy. The fuzzy safety level is now assessed by comparing the resulting fuzzy set of reliability index with the value of 'required' reliability index. The proposed hybrid approach combines the advantage of taking cognizance of the information content in probability distributions and the inherent possibility levels in fuzzy variables.

Let the performance function be represented by f (P_1, P_2, ..., P_n, F_1, F_2, ..., F_m), where P_1, P_2, ..., P_n are 'n' model parameters which are random variables and may be adequately represented by PDFs and F_1, F_2, ..., F_m are 'm' other model parameters, which are fuzzy variables and may be represented by fuzzy numbers. The hybrid approach (Venkatachalam et al.,

2004) is based on combining the RPEM technique with the method of α-cuts. Here it is assumed that the PDFs are already known. Otherwise, PDFs are to be either determined from adequate data or to be generated by Monte Carlo Simulation technique. The proposed approach may be explained algorithmically as follows:

1. First consider the random variables P_1, P_2, ..., P_n. Get 2^n points from the 'n' PDFs as per RPEM, namely (Fig.5a).

$$P_{1-} = \mu_{P_1} - \sigma_{P_1}$$

$$P_{1+} = \mu_{P_1} + \sigma_{P_1}$$

$$P_{n-} = \mu_{P_n} - \sigma_{P_n}$$

$$P_{n+} = \mu_{P_n} + \sigma_{P_n}$$

2. Now consider the fuzzy variables F_1, F_2, ..., F_m. Select a a value of the membership function. Then select the end points of the a-cut of fuzzy variables according to vertex method (Fig.5b) ,i.e. F_{1-}, F_{1+}, ..., F_{m-}, F_{m+}.

3. The combinations of end points constitute different extreme scenarios of the fuzzy variables thereby giving 2^m combinations where 'm' is the number of fuzzy variables.

4. Calculate Reliability index at each a level by considering 2^n vertices for PDFs using RPEM for each of the 2^m combinations of fuzzy variables, thereby resulting in 2^m reliability indices at each α level. Suppose performance function is $f(P_1, P_2 \text{ and } F_1)$ then various 2^m (here $m = 1$) combinations of the parameters are [(P_{1+}, P_{2+}, F_{1-}), (P_{1+}, P_{2-}, F_{1-}), (P_{1-}, P_{2+}, F_{1-}), (P_{1-}, P_{2-}, F_{1-})] and [(P_{1+}, P_{2+}, F_{1+}), (P_{1+} ,P_{2-}, F_{1+}), (P_{1-}, P_{2+}, F_{1+}), (P_{1-}, P_{2-}, F_{1+})]. Using these combinations $E[FS]$ and $\sigma[FS]$ are calculated by RPEM using Eqns.37 and 38 and then 2^m reliability indices are calculated using Eqn.39.

$$E[FS] = p_{++}FS_{++} + p_{+-}FS_{+-} + p_{-+}FS_{-+} + p_{--}FS_{--} \tag{37}$$

$$\sigma[FS] = \sqrt{E[FS^2] - (E[FS])^2} \tag{38}$$

$$\beta = \frac{E[FS]-1}{\sigma[FS]} \tag{39}$$

where p_{++} is the weight factor and corresponding FS at point $(\mu_{P_1} + \sigma_{P_1},\ \mu_{P_2} + \sigma_{P_2})$

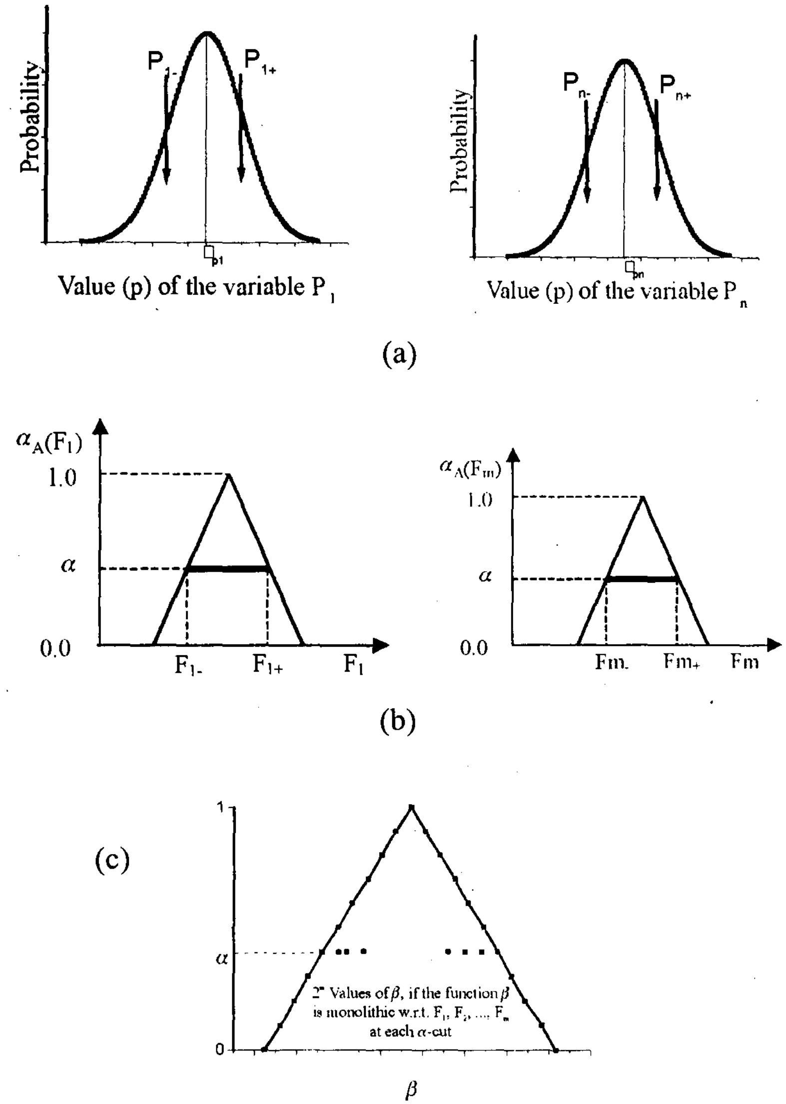


FIGURE 5 : Schematic Illustration of the Proposed Hybrid Approach

5. This procedure is repeated for all a levels. Build the fuzzy reliability index by selecting the inferior and superior values of reliability index at each α level (Fig.5c). The fuzzy reliability index β obtained by selecting maximum and minimum values of β is given in Fig.6. where $\beta_{(\alpha=0),l}$ is the lower bound (left side) value of β at zero alpha value and $\beta_{(\alpha=0),r}$ is the upper bound (right side) value of β at zero alpha value in the fuzzy reliability index $\tilde{\beta}$.

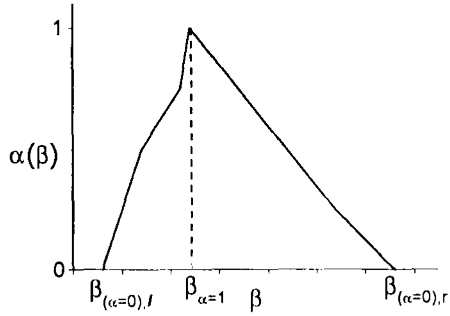


FIGURE 6 : Fuzzy Reliability Index β

Validation of Suggested Methods

Reliability analysis primarily concerns itself with establishing the confidence that can be reposed on the deterministic factor of safety. One of the handicaps in validating, therefore, is the inherent difficulty in measuring or perceiving the confidence level in the field. The common way to validate is to analyze a failed slope and demonstrate that reliability analysis gives a better explanation for the observed failure (Duncan, 2000). Failed natural slopes are well suited for this, since they often have a FS close to unity under normal conditions, but a high probability of failure.

Application to Natural Slopes

The methods described above have been applied to evaluate the stability of a natural slope. The problem has been analyzed by four reliability methods namely, Rosenblueth's Point Estimate Method (RPEM), First Order Second Moment method (FOSM) and the newly proposed Fuzzy Point Estimate Method (FPEM) and Fuzzy Random Probabilistic Method (FRPM). In all the methods reliability index and probability of failure are computed by assuming normal distribution of FS.

Case Study of an Infinite Slope

To begin with, the example of an infinite slope analysis is used. A slope that extends for a relatively long distance and has a consistent subsoil profile may be analyzed as an infinite slope. The failure plane for this case is parallel to the surface of the slope.

This case study relates to a landslide, which occurred on June 28, 1994 in the village of Parmachi (Latitude 18° 8′N and Longitude 73°36′E) in the Western Ghats, located about 300 km south of Mumbai. Prior to the

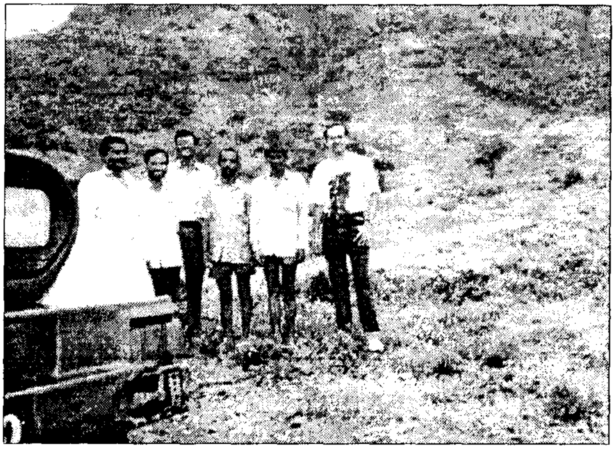


FIGURE 7 : A Landslide at Parmachi, Western Ghats

occurrence of the landslide, there was a heavy downpour in the area for three days with a maximum intensity of 240 mm/day. The landslide involved movement of about 30,000 m^3 of soil and rock over approximately 300 m (Fig.7).

For analysis, a long natural slope of infinite extent is considered. Although the slope angle varies over a wide range, an angle of 40° to the horizontal is considered for purposes of effective illustration of reliability analysis. The water table is below the potential failure surface in normal conditions. A slip surface has developed on a plane parallel to the surface at a depth of 1.5 m. The total thickness of overburden to the bedrock is about 3 m. The slope and soil data used for the analysis is given in Table 3. Here,

c = effective soil cohesion (kN/m^2);

γ_{sat} = saturated unit weight of the soil (kN/m^3);

TABLE 3 : Slope and Soil Data used for Parmachi Landslide

Slope and Soil Parameters	Mean	Range	COV (%)
z (m)	1.5	0.5 to 3.00	60
α (degrees)	40	40 to 60	20
k (m/hr)	3.5×10^{-3}	3.8×10^{-4} to 0.035	50
ϕ' (degrees)	30	25 to 36	10
c' (kN/m^2)	10	4 to18	40
γ (kN/m^3)	21.7	20.3 to 22.74	3
η	0.423	0.330 to 0.550	20

γ_1 = unit weight of the soil (kN/m^3);
H = thickness of soil cover (m);
h = groundwater table height (m);
z = depth of the failure surface from the top (m);
ϕd = angle of friction (degrees);
β = inclination (degrees); and
η = porosity.

Reliability of the slope is analysed by considering the variabilities in c′, ϕ' and depth of failure surface (z) and the fluctuation in ground water table. Two different correlations are considered among the variables. All the four reliability methods have been used for the reliability analysis.

Analysis by Four Reliability Methods

Deterministic stability analysis by Bishop's simplified method shows that the slope is marginally safe under dry conditions with a factor of safety (CFS) of 1.312.

Two different correlations considered are:

i) Set 1: ρ (c′, ϕ') = 0.25, ρ (ϕ', z) = 0.40 and ρ (z , c′) = 0.20; and

ii) Set 2: ρ (c′, ϕ') = 0.25, ρ (ϕ', z) = 0.40 and ρ (z, c′) = 0.30

Rosenblueth's Point Estimate Method and First Order Second Moment Method

The values of the uncertain parameters at different α-levels are calculated as shown in Table 4. The results of the analysis by RPEM and FOSM are given in Table 5.

TABLE 4 : Uncertain Parameters at α-Levels

α Level	Uncertain Parameters							
	c′+ (kN/m^2)	c′− (kN/m^2)	ϕ'+ (degrees)	ϕ'− (degrees)	γ+ (kN/m^3)	γ− (kN/m^3)	z+ (m)	z− (m)
0.2	14.8	5.2	33.6	26.40	22.48	20.92	2.580	0.420
0.4	13.6	6.4	32.7	27.30	22.285	21.115	2.310	0.690
0.6	12.4	7.6	31.8	28.20	22.09	21.31	2.040	0.96
0.8	11.2	8.8	30.9	29.1	21.895	21.505	1.770	1.230

TABLE 5 : Variation of E[FS], σ[FS] and p_f with Fluctuation in Groundwater Level (c′, ϕ' and z as Random Variables) (RPEM and FOSM) [ρ (c′, ϕ') = 0.25, ρ (ϕ', z) = 0.4 and ρ (z, c′) = 0.20]

z_w/z	CFS	E [FS]		σ [FS]		p_f	
		RPEM	FOSM	RPEM	FOSM	RPEM	FOSM
0.0	1.311	1.610	1.311	0.663	0.569	0.179	0.293
0.2	1.244	1.546	1.244	0.657	0.564	0.203	0.336
0.4	1.178	1.475	1.178	0.651	0.561	0.233	0.378
0.6	1.113	1.406	1.113	0.646	0.556	0.265	0.423
0.8	1.049	1.338	1.049	0.641	0.553	0.299	0.471
1.0	0.986	1.271	0.986	0.636	0.549	0.335	0.514

Suggested Fuzzy Point Estimate Method

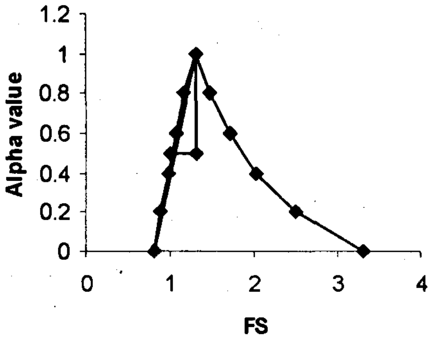
In FPEM method the variables c′, ϕ' and z are considered as fuzzy variables. These are fuzzified with k = 1.5. Analysis using FPEM is carried out for all m (= z_w/z) values and shown in Table 6. The E [FS] is slightly increased at all 'm' values compared to FOSM results, but σ [FS] has been decreased drastically. Due to significant deciease in the standard deviation, the probability of failure is very less, when compared to the other reliability methods. No correlation effect is considered in this method.

Suggested Fuzzy Random Probabilistic Method

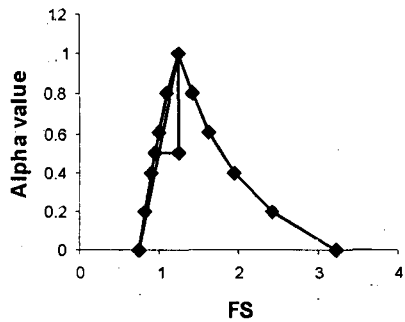
Here the variables c′, ϕ' and z are considered as fuzzy random variables. The variables are fuzzified using k = 1.0, and then the effect of randomness is combined with it. Then FRPM is used to evaluate the lower and upper bounds of FS at each α-level, which results in one FS fuzzy set at each 'm', totally 11 FS fuzzy sets. Typical ones are given in Figs.8a to 8f. From the resulting fuzzy sets in Fig.8, the equivalent value of FS for FS

TABLE 6 : Variation of E [FS], σ [FS] and p_f with Fluctuation in Groundwater Level (c′, ϕ' and z as Fuzzy Variables) (FPEM)

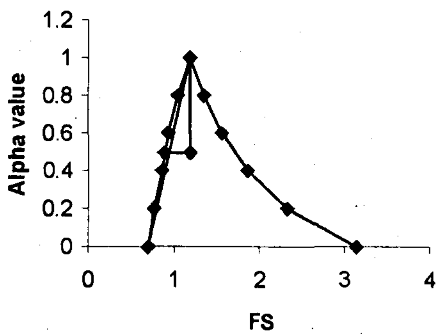
z_w/z	E [FS]	?[FS]	p_f
0.0	1.362	0.117	0.00135
0.2	1.293	0.119	0.0069
0.4	1.227	0.120	0.02938
0.6	1.161	0.122	0.09358
0.8	1.097	0.124	0.2171
1.0	1.034	0.126	0.39358



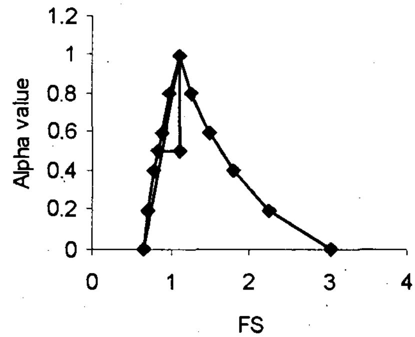
(a) FS Fuzzy Set (m = 0.0)



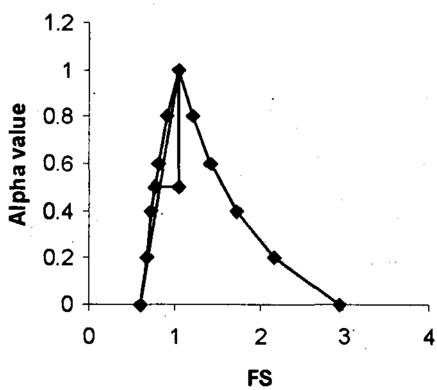
(b) FS Fuzzy Set (m = 0.2)



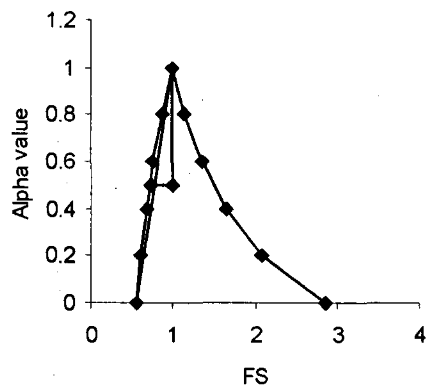
(c) FS Fuzzy Set (m = 0.4)



(d) FS Fuzzy Set (m = 0.6)



(e) FS Fuzzy Set (m = 0.8)



(f) FS Fuzzy Set (m = 1.0)

FIGURE 8 : FS Fuzzy Sets

TABLE 7 : Equivalent Value of FS from FS Fuzzy Sets

Value of 'm'	FS_{min} at $\alpha = 0.7$	Average of FS_{min} in $0.5 < \alpha < 1.0$
0.0	1.118	1.182
0.2	1.055	1.114
0.4	0.992	1.050
0.6	0.931	0.992
0.8	0.871	0.930
1.0	0.812	0.870

fuzzy set possessing a reasonable degree of belief may be taken as one of the following:

- average of all lower bound values in the range $0.5 < \alpha < 1.0$
- lower bound value of FS at $\alpha = 0.7$
- The centriod of the triangle ABC.

Two of the typical equivalent values are given in Table 7.

Two typical curves of variation of reliability of index with α-value are shown in Fig.9a and 9b. From m = 0 to m = 0.8, the reliability index increases as α-value increases. It means probability of failure increases as α-value decreases.

Spatial Variation of Failure Probability In DEM and GIS Environment

Although, for convenience, a single value of failure probability is usually associated with a slope, in a real situation, the failure probability itself would vary within the slope. A method to evaluate the spatial variation using Digital Elevation Model (DEM) and Geographic Information System (GIS) concepts (Venkatachalam et al., 2002) is proposed here. The example of the same Parmachi landslide is used to illustrate. The results are presented for random variables and probabilistic analysis.

Background

The need for spatial analysis is increasing with hilly areas witnessing unprecedented growth in recent years. Occurrence of landslides along long stretches of hill roads is a common sight during heavy monsoon. As a consequence, infrastructural lifelines such as communication and road network are under stress.

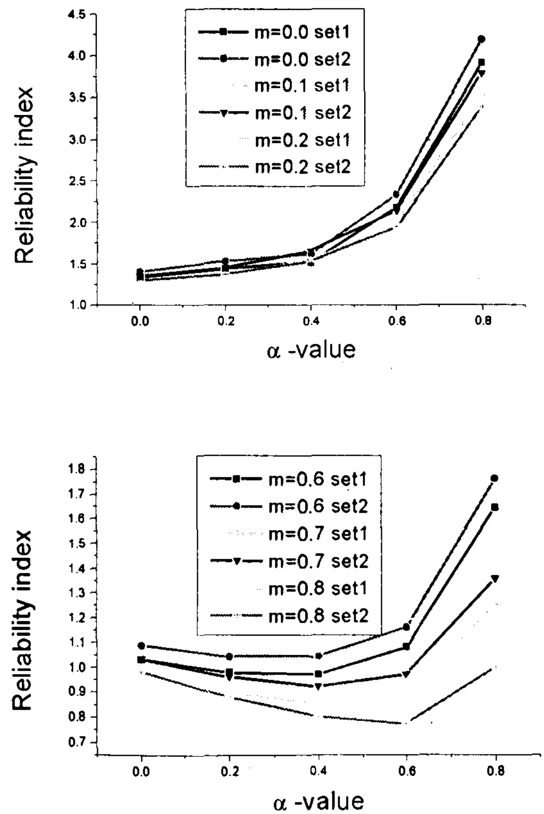


FIGURE 9 : Variation of Reliability Index with α-Value

Study of rainfall induced slope failure and landslides has been a topic of special interest in many countries experiencing tropical rainfall. The obvious way to predict failure would be to establish statistically (Au, 1993) or numerically correlations between rainfall parameters such as intensity, hourly or daily rainfall and duration. The importance of antecedent moisture is considered by Ng and Shi (1997). A mechanistic approach involving study of matric suction and its gradual reduction during rainfall leading to slope failure is looked into by Anderson and Sitar (1995). Recently, Babu and Murthy (2005) have presented reliability analysis of unsaturated slopes.

A later development was the incorporation of the infinite slope concept in a raster GIS (Sakellariou and Ferentinou, 2001). However, a more versatile solution developed earlier by (Nair et al., 1996) is used in the present example.

Methodology used

The methodology adopted here (Venkatachalam, 2003) involves building a DEM first. For this, the study area of about 500 m × 400 m extent has been divided into grids of 16 m × 16 m size. The hill where the landslide occurred has elevations varying from about 240 m to 500 m. The slopes vary from 50° to 60°. The soil is residual and the cover varies from 0.5 m to as much as 10 m. A stability analysis has been carried out for different intensities and durations of rainfall using the infinite slope method. This method has been used since it allows superposition of thematic information, especially for subsequent risk evaluation. A factor of safety value is obtained for each grid using Eqn.40.

$$F = \frac{c_d + \left[(\gamma_{sat} - \gamma_w)(h - z) + \gamma_t (H - h)\right]\cos^2\beta \tan\phi_d}{\left[\gamma_{sat}(h - z) + \gamma_t (H - h)\right]\cos\beta \sin\beta} \tag{40}$$

where the parameters are same as those used in Table 3. The time to failure is calculated on the basis of h required for the factor of safety to become unity as:

$$t = \frac{\eta}{\varepsilon \sin\beta \cos\beta}\left[-1 + \left(\sqrt{1 + \frac{2h}{r}\varepsilon k \sin\beta \cos\beta}\right)\right] \tag{41}$$

The fluctuation of the groundwater table, the effect of vegetation, in the form of root cohesion c_t and tree weight T are incorporated. The thematic information required for this is obtained from different sources. Remote sensing is useful in obtaining information about vegetation and landuse and the variable soil conditions. The results are presented in Fig.10. Figure 10a shows the spatial variation of failure probability. By choosing a threshold probability the area most prone to failure is identified as shown in Fig.10b. This clearly shows that a rainfall of 240 mm/day (0.01 m/hr) leads to a large patch of contiguous grids failing, indicating failure of the slope, i.e. occurrence of a landslide.

Validation Using Centrifuge Modelling

In order to validate this, studies have been conducted on a small centrifuge available at IIT Bombay. The specifications of this centrifuge are:

Type	– Swinging buckets on either side of the arm

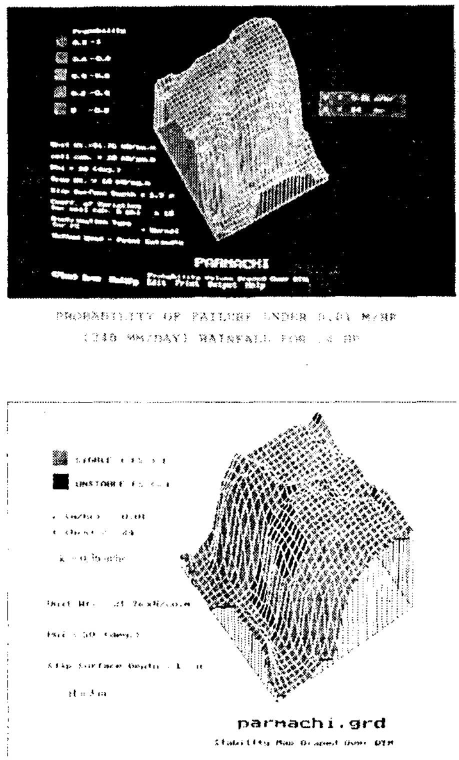


FIGURE 10 : Spatial Variation of Failure Probability and Simulation of Parmachi Landslide

Speed of the centrifuge	– 960 rpm
Arm radius	– 20 cm
Maximum pay load	– 2.4 kg
Maximum acceleration	– 300 g
Capacity	– 0.72 g-tons
Maximum Depth of the sample	– 5.0 cm

Rainfall is produced by a special arrangement for spraying water on the slope model during flight. Simulation of rainfall is done in terms of drop size, depth of fall, terminal velocity and intensity of rainfall. In the centrifuge the drop size is about 50 to 60 microns, the fall is 4 cm to 11 cm. At a g value of 100 these will correspond to those in the prototype. The intensity was measured by collecting the water during the test and measuring the volume. Model tests were conducted using Powai silt under different sample sizes, slope angles, initial moisture contents, depths of fall of rain, intensity and g values (Quadri, 2000). Tests were conducted upto failure and rainfall duration was measured. The results are presented in Fig.11.

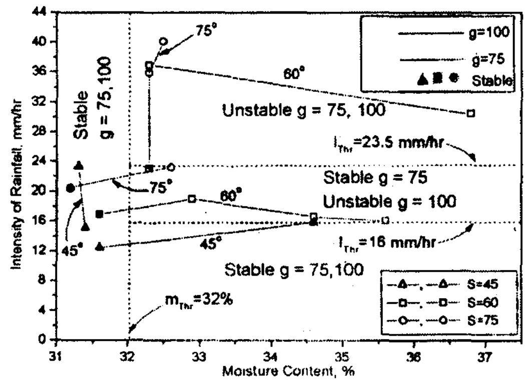


FIGURE 11 : Rainfall Effect on Slope Stability

Although qualitatively, these results demonstrate a few important aspects:

(i) the existence of a threshold antecedent moisture content upto which the slopes are stable irrespective of the rainfall intensity,

(ii) that different slopes fail at different threshold rainfall intensities, and

(iii) the threshold rainfall intensity at which high slopes are stable is less than 16 mm/hr. (380 mm/day).

This strongly suggests that the failure of the Parmachi slope at an intensity of 240 mm/day was but inevitable, as indicated by the probability of failure obtained in the reliability analysis.

Risk Evaluation

The main purpose of a stability analysis of this type is to predict the possible mass-movement and run-out and the resulting risk. The run-out of a typical debris flow during a landslide depends on momentum transfer. The special feature in a landslide is that, both the sliding mass and the velocity continuously decrease as deposition progresses. There are several mass movement models available for representing this phenomenon. Here a fractal deposition model is evolved and used (Venkatachalam, 1995).

Fractal Mass-movement Model

The deposited mass M_d and distance travelled along the paths have a definite relationship. Two non-dimensional terms are calculated:

(i) the term M_p/M_i, where M_i is the initial displaced mass and M_p is mass participating in movement (= $1-M_d$) at a location at distance s, and

(ii) the term $(s_f - s)/s_f$.

A power law relationship for deposition pattern is derived from known deposition profiles of a number of landslides (Quadri, 2000). To compute M_i, the critical slip surface is located by a deterministic technique.

Analysis of deposition profiles of a number of landslides has shown that a power law relationship holds. The following relations are now derived in terms of the fractal dimension **D** of the terrain from fundamental principles of mechanics.

For run-out distance:

$$s_f = -\frac{v_0^2}{A}\{2\times(2.6479-1.15437\times D)+1\}+s_i \quad (42)$$

where A = $2g(\sin\beta - \mu\cos\beta)$

For deposition profile:

$$\frac{dM}{ds} = -\frac{M_i}{s_f}\times 1.4\times(2.6479-1.15437\times D)\times\left(\frac{s_f - s}{s_f}\right)^{1.6479-1.15437\times D} \quad (43)$$

The deposition profile of the Parmachi landslide is computed and presented in Table 8. This agrees well with the observed values. The volume of displaced soil and rock was approximately 30,000 m^3 and the run-out was about 300 m.

TABLE 8 : Deposition Profile

Distance (m)	Moving Mass (cu. m)	Thickness of Deposition (m)
100	294.19	0
150	206.38	1.75
200	127.96	1.57
250	61.25	1.33
275	33.62	1.11
300	11.37	0.89
321	0	0

Computation of Risk

The definition of risk given above is used in the present study. Traditional deterministic methods of analysis may be supplemented by probabilistic analyses. In the former, the safety factor FS is used as the index of susceptibility to failure. But, lower FS doesn't necessarily mean higher proneness to failure. Therefore, the probability of failure p_f or the reliability index β is a better measure. Table 9 shows the probabilities of all likely events (Venkatachalam et al., 2003) for a critical rainfall of 240 mm/day (0.01 m/hr). The alternative with the least expected cost is chosen if the expected value is the criterion for decision. Thus the evaluation of risk could help in deciding and planning suitable remedial or preventive measures.

The total probability of occurrence of the catastrophic event works out to 0.327, while the cumulative or overall probability of damage to structures works out to 0.228. It is necessary to consider these together in order to interpret them properly.

According to (Chowdhury and Flentje, 2003), it is important to consider the severity of the consequences associated with a hazard event in addition to its probability of occurrence. There are high-probability, low-consequence events as well as low-probability, high-consequence events. Events with low probabilities are subject to greater uncertainty than that of high probability events. Hence, a hazard-consequence matrix has to be developed and several risk categories are to be considered and system reliability has to be evaluated. Thus, the risk level of this slope can be classified as 'medium', and an appropriate method of stabilization can be chosen accordingly.

Role of Remote Sensing and GIS

The role of remote sensing is primarily to assist in landuse-landcover mapping and to correlate the propensity to landsliding with likely losses.

TABLE 9 : Probability Table

Chance Node	Event	Event Probability	Cumulative Probability
1	Critical rainfall of 240 mm/day	0.70	
2	Landslide	0.467	0.327
3	Damage to:		
	– structures	0.70	0.228
	– Loss of life	0.20	0.065
	– Other losses	0.10	0.033

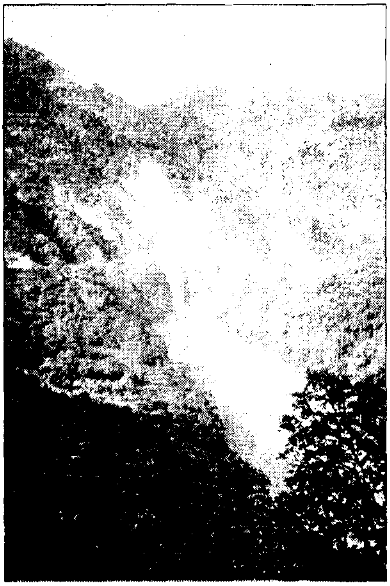


FIGURE 12 : View of Kaliasaur Landslide

GIS, being a spatial analysis tool, can assist in spatial data manipulation gathered by remote sensing and other methods. Their usefulness can be enhanced with the help of digital techniques such as the Optimal Rotational Transformation Technique (Venkatachalam and Jeyasingh, 1986; Nirala and Venkatachalam, 2000) for choosing the most suitable band combinations for a given purpose. The Shortest Path Technique of GIS, which is available in most GIS packages, can be adapted (Nagesha and Venkatachalam, 2000) for locating the critical failure surface – the 'shortest' (least) factor of safety surface – of a slope.

Case Study of a Finite Slope

The case study of the Kaliasaur landslide is considered based on data from (CBRI, 1988). This landslide occurred on 19th September 1969 at 147 km post on H-45 on Hardwar-Badrinath road and was situated in a sharp bend on the left bank of Alaknanda river (Fig.12). It blocked a greater part of the width of the river flowing about 100 m below the road level. Road was badly damaged over a stretch of 300 meters. The slide mass consisted of soil derived from highly weathered quartzite. Detailed studies have been carried out and reported in CBRI (1988). These have been used. The section used for the analysis is given in Fig.13. After studying the data presented, based on the variations observed and also on the basis of observations reported by Duncan (2000) COVs of 25, 10, 3 and 50 percent have been adopted as shown in Table 10.

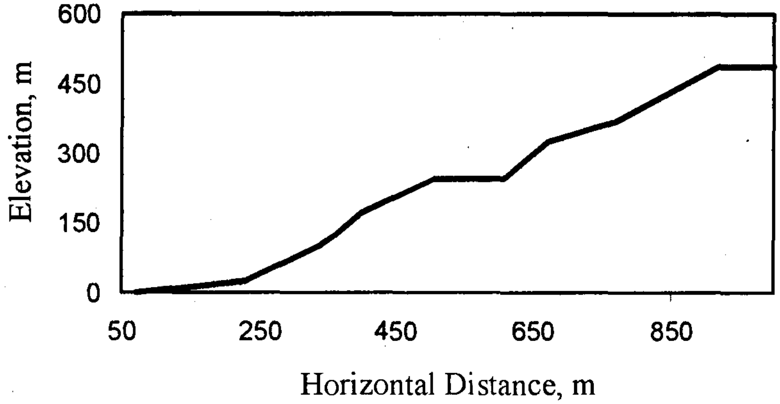


FIGURE 13 : Kaliasaur Slope Section

TABLE 10 : Kaliasaur Slope Data*

Soil parameters	Mean	COV (%)
c (kN/m^2)	100	25
ϕ (degrees)	30	10
γ (kN/m^3)	27	3
r_u	0.20	50

* after CBRI (1988)

Results of Reliability Analysis

By Bishop's simplified method the Central Factor of Safety (CFS) of the slope is obtained as 1.141 under normal conditions.

Rosenblueth's Point Estimate Method

For the analysis by RPEM, the parameters c, ϕ and r_u are considered as random variables. The results are presented in Table 11. Three different correlations [ρ (c, ϕ) = 0, 0.25 and −0.50] are considered between c and ϕ. The value of E [FS] is 0.883 for zero correlation between strength parameters c and ϕ. The probability of failure comes out to be 0.763. Thus, reliability analysis offers an explanation for the occurrence of the landslide.

First Order Second Moment Method

The results of FOSM are also given in Table 11. From the results it can be seen that the E [FS] and σ [FS] are almost same as that of RPEM method. The probability of failure comes out to be 0.766.

TABLE 11 : Reliability Analysis by RPEM and FOSM

ρ (c, ϕ)	0.0		0.25		0.50	
	RPEM	FOSM	RPEM	FOSM	RPEM	FOSM
E [FS]	0.883	0.882	0.884	0.882	0.883	0.882
σ [FS]	0.164	0.161	0.171	0.169	0.149	0.146
β	−0.705	−0.728	−0.675	−0.697	−0.779	−0.728
p_f	0.763	0.766	0.750	0.757	0.782	0.767

Fuzzy Point Estimate Method

In this method each of the fuzzy variables is assumed as a triangular fuzzy number (TFN) and discretised into ten α-levels with their corresponding intervals. The α-level intervals for each uncertain parameter are given in Table 12. In order to demonstrate the usefulness of reliability analysis, value of k is taken as 1.0, although the well known three-sigma rule could have been applied. The interval values for FS are also presented in Table 12. The results of the FPEM method are presented in Table 13. The probability of failure is 0.919, which shows an increase due to the consideration of fuzziness in the input parameters. This points to the criticality of the epistemic uncertainties, which are characteristic of geotechnical engineering.

Fuzzy Random Probabilistic Method

In FRPM method, the mean values of the input parameters (c, ϕ and

TABLE 12 : The α-Level Intervals for Uncertain Parameters c, ϕ and r_u and Resulting Fuzzy FS Set

α-level	Uncertain Parameters						Fuzzy FS Set FS^-/FS^+
	c+	c−	ϕ+	ϕ−	r_u+	r_u−	
0.1	122.5	77.5	32.7	27.3	0.29	0.11	0.863 / 0.880
0.2	120.0	80.0	32.4	27.6	0.28	0.12	0.866 / 0.882
0.3	117.5	82.5	32.1	27.9	0.27	0.13	0.869 / 0.883
0.4	115.0	85.0	31.8	28.2	0.26	0.14	0.872 / 0.883
0.5	112.5	87.5	31.5	28.5	0.25	0.15	0.874 / 0.884
0.6	110.0	90.0	31.2	28.8	0.24	0.16	0.876 / 0.884
0.7	107.5	92.5	30.9	29.1	0.23	0.17	0.878 / 0.884
0.8	105.0	95.0	30.6	29.4	0.22	0.18	0.880 / 0.884
0.85	103.75	96.25	30.45	29.55	0.22	0.19	0.881 / 0.884

TABLE 13 : Results of FPEM

E [FS]	σ [FS]	Overall β	p_f
0.870	0.092	− 1.4	0.919

TABLE 14 : The α-Level Intervals for Fuzzy Mean Values of c, ϕ and r_u

α-level	Uncertain Parameters					
	c+	c−	ϕ+	ϕ−	r_u+	r_u−
0.0	127	73	33.24	26.76	0.308	0.092
0.2	121.6	78.4	32.592	27.408	0.286	0.114
0.4	116.2	83.8	31.944	28.056	0.264	0.135
0.6	110.8	89.2	31.296	28.704	0.243	0.1568
0.8	105.4	94.6	30.648	29.352	0.221	0.178
0.9	102.7	97.3	30.324	29.676	0.2108	0.189

r_u) are considered as fuzzy, whereas their variances are considered non-fuzzy. In general, however, both would be fuzzy when the input parameters are fuzzy. Though, in principle, both can be treated as fuzzy, it is observed that the former is usually adequate.

The input parameters are again fuzzified using k = 1.0, then the effect of randomness is combined with these parameters. The α-levels and the intervals for the fuzzy mean values are given in Table 14.

At each α-level, there will be an upper bound (FS_{max}) and lower bound (FS_{min}) values of FS, thus resulting in a FS fuzzy set, which is shown in Fig.14.

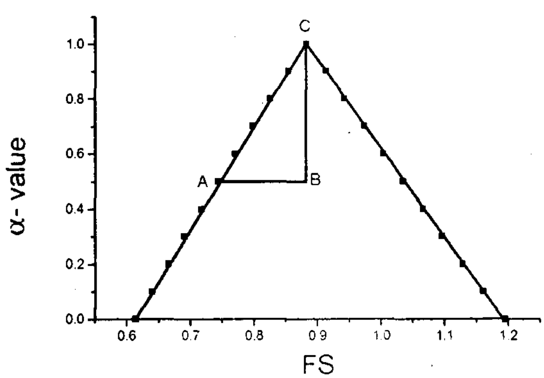


FIGURE 14 : Fuzzy set of Factor of Safety

TABLE 15 : Values of E [FS[at Different α-values

α-Level	E [FS]		
	ρ (c, ϕ) = 0.0	ρ (c, ϕ) = −0.5	ρ (c, ϕ) = 0.25
0.0	0.883	0.851	0.899
0.2	0.884	0.857	0.897
0.4	0.885	0.864	0.895
0.6	0.8828	0.869	0.889
0.8	0.883	0.876	0.886
1.0	.882	0.882	0.882

The values of E [FS] at all selected α-levels are given in Table 15. Three correlations [ρ (c, ϕ) = 0.0, 0.25 and −0.50) are considered between c and ϕ. For the positive correlation the E [FS] decreases as α-level increases, whereas for negative correlation it increases as α-level increases.

The values of probability of failure and reliability index at each of the selected α-level are evaluated. The variation in β with α-values are given in Fig.15. The plots of reliability index with α-levels shows that, as the value of a increases the reliability index decreases implying that higher belief or confidence can only be had at a lower value of β. As the correlation between strength parameters is more towards positive, the decrease in the value of β is observed at all the α-levels.

From the resulting fuzzy set in Fig.14, the equivalent value of FS for FS fuzzy set can be taken, as already explained, as follows:

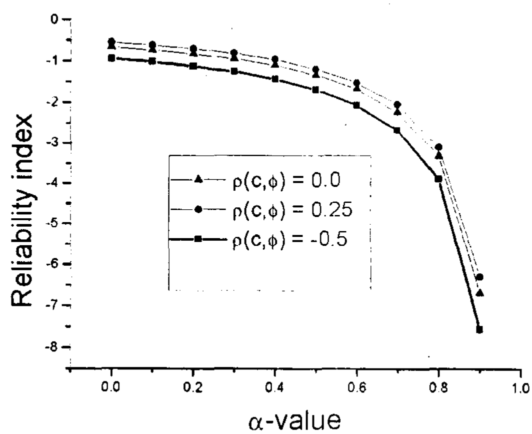


FIGURE 15 : Dependence of Reliability Index on α-value

- Equivalent value of FS for fuzzy FS set is the average of all lower bound values in the range $0.5 < \alpha < 1.0$ and found to be 0.826.
- The equivalent value of FS for fuzzy FS is the lower bound value of FS at $\alpha = 0.7$ and it comes out to be 0.798.
- The centroid of the triangle ABC of Fig.14 is 0.835, which can also be taken as equivalent FS value for FS fuzzy set.

If these values differ significantly, unlike this example, the minimum of the three would be the best choice.

In all the cases the FS is less than 1. The value of β by all the three reliability methods is less than 0.5; hence the slope is highly unstable. The stability analysis with zero pore pressure shows that the slope is marginally safe (i.e. FS = 1.141), but the uncertainty in the strength parameters and rise in pore pressure are leading to the failure.

Although RPEM does not consider derivatives of performance function it gives results, which are comparable to FOSM method, with the advantage of fewer computations. Fuzzy variables with resulting random FS give a lower value of reliability index. Treating the variables as random, (but the resulting FS as fuzzy) gives a whole range of reliability indices for the various α-levels. If an average value of α-cut corresponding to a desired β is required it may be taken from the linear portion of the curve between reliability index and α-value. However, further studies are required to identify what may be considered an appropriate α-value for reliability-based design, in any given instance.

The values of β for the Parmachi and Kaliasaur landslides by RPEM, FOSM and FPEM methods are low and negative. The corresponding p_f values are also high. This clearly explains why the two slopes failed under the given conditions, even though their conventional critical FS values are 1.312 and 1.141, respectively.

TABLE 16 : Summary of Results

Case Study	FOSM		RPEM		FPEM		FRPM	
	(β)	(p_f)	(β)	(p_f)	(β)	(p_f)	(β)	(p_f)
Parmachi Landslide (considering rainfall)	−0.050	0.531	−0.075	0.529	−0.075	0.529	4.185 to 0.153	0.145×10^{-4} to 0.44
Kaliasaur Landslide	−0.728	0.766	−0.779	0.780	−1.40	0.919	−0.551 to −7.57	0.708 to 0.999

The minimum reliability index (β) and corresponding probability of failure (p_f) for the two case studies are summarised in Table 16 for all the different types of uncertainties and the suggested methods.

The values of p_f by FPEM are higher since the fuzziness in the data is also accounted for. RPEM gives a whole range of p_f, since β itself is fuzzy and has an interval.

FS vs β and p_f – How does one interpret?

The Reliability Index obtained by the above analyses needs to be interpreted carefully. The main advantage in using reliability analysis is that it can account for uncertainties unlike the deterministic analysis. Guidelines for interpreting β are not many. Kulhawy and Phoon (2002) and US Army (1999) opine that a β of about 1.0 or less corresponds to a p_f of 0.16 or more and indicates a catastrophic situation. The above results are in good agreement with that. Also, a β of at least 3.0 is considered to indicate reasonably acceptable performance. Chowdhury and Flentje (2003) also suggest a minimum β of 3.0 for such slopes. Hence, these natural slopes need to be stabilized with strengthening measures which would give a β of at least 3.0. Another consideration is the risk and the consequent damage. Hence, the value of β would be even higher in urban settings, where the consequences are likely to be higher. However, more experience in implementation and validation of reliability-based design is required before these guidelines can be modified.

Critical Deterministic vs. Minimum Reliability Surface

The surface with minimum factor of safety may not necessarily be the surface with minimum reliability (Hassan and Wolff, 1999); hence there is a need to search for the surface with minimum reliability.

The surface of minimum FS and the surface of minimum β do not generally coincide. The reliability index of the critical deterministic slip surface for a slope is known as β_{det}, but this may not necessarily be the minimum value of β. The minimum reliability index β_{min} and critical probabilistic surface can be located using 2^n combinations of the soil parameters where n is the number of variables. The different β values can be defined as:

1. β_{det} is the reliability index of the slope corresponding to the deterministic slip surface {i.e. the value of reliability index that is corresponding to Central Factor of Safety (CFS)}

2. β_{min} is the reliability index corresponding to minimum reliability surface

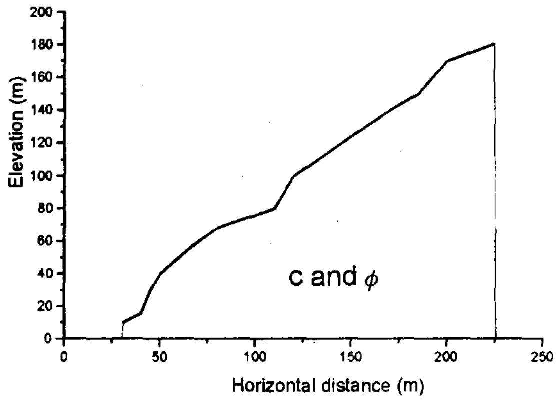


FIGURE 16 : Mussoorie Slope Section

(i.e. the surface with lowest reliability index / highest probability of failure)

3. $\beta_{overall}$ is the reliability index associated with the combination of all surfaces ($\beta_{overall}$ is calculated using reliability method such as RPEM, FOSM or FPEM).

In this section critical probabilistic surface of slopes is identified using an approach based on RPEM to search for the critical probabilistic surface utilizing existing deterministic slope stability programs. From design considerations, β_{min} would be the appropriate parameter, unless there are several modes of failure. For a homogeneous slope, it has been seen that both β_{det} and β_{min} values are close and the corresponding surfaces are also close to each other.

Case Study of a Finite Slope

A case study of a landslide in Mussoorie has been considered (CBRI, 1988) as an example for safety verification using the hybrid approach. This landslide was located at 5 km on Mussoorie bypass road. The area suffered a major landslide in 1983, which damaged a number of buildings located on the slope.

The slide area was highly unstable due to accumulation of colluvium, debris and moderate to highly weathered rock. The causative factors of the landslide were extensive toe erosion, toe removal and heavy precipitation. The slide movement involved rock fall and debris slide. A section along the slope has been considered for the analysis with the given soil properties and pore pressure value. The section used for the analysis is given in Fig.16. The soil data used for the analysis is given in Table 17.

TABLE 17 : Mussourie Slope Data

Soil parameters	Mean	COV (%)
c (kN/m^2)	100	20
Φ (degrees)	30	10
γ (kN/m^3)	27.5	3
r_u	0.20	50

The CFS is 1.1 under normal conditions as mentioned earlier. Parameters c, ϕ and r_u are considered as random variables for the determination of minimum reliability surface of the slope. The minimum reliability surface is obtained by considering all possible combinations of c', ϕ', ... unlike Hassan and Wolff (1999). The values of β_{det}, β_{min} and $\beta_{overall}$ are calculated and presented below along with the corresponding FS values and slip circle parameters.

(i) From the CFS value β_{det} is calculated as mentioned earlier. CFS value is 0.893 and corresponding β_{det} is -0.6721. The slip circle parameters of critical deterministic surface are X = 80.00 m, Y= 229.25 m and R= 152.189 m.

(ii) β_{min} is calculated as the least of all the 2^n values of FS and corresponding β which are obtained according to RPEM. The value of least factor of safety is 0.633. The slip circle parameters of this minimum reliability surface are X = 2.0 m, Y= 242.50 m and R = 234.48 m. β_{min} will correspond to this surface.

(iii) $\beta_{overall}$ is calculated using RPEM from the value of E [FS] as usual. Its value is 0.867 and corresponding $\beta_{overall}$ is found to be -0.6925.

Table 18 gives the values of FS_i for all the parameter combinations.

TABLE 18 : Factors of Safety [for ρ (c, ϕ) = 0.0]

Slip surface	FS_i
1	0.884
2	1.173
3	0.633*
4	1.003
5	0.654
6	1.047
7	0.650

* Minimum reliability

TABLE 19 : Values of β_{det}, β_{min} and $\beta_{overall}$

ρ (c, ϕ)	β_{det}	β_{min}	$\beta_{overall}$	Minimum reliability surface			Critical deterministic surface		
				X (m)	Y (m)	R (m)	X (m)	Y (m)	R (m)
0.0	−0.672	−2.305	0.692	2.00	242.5	234.48			
−0.50	−0.74	−2.538	0.775	2.00	242.5	234.48	80.00	229.25	152.18
0.25	−0.644	−2.210	0.650	2.00	242.5	234.48			

The results for three different correlations between strength parameters i.e. ρ (c, ϕ) = 0.0, 0.25 and −0.50 are given in Table 19. In this case the probability of failure is found to increase for decreasing correlations. This is due to the very low values of factor of safety (FS).

System Reliability

The calculation of minimum factor of safety or the minimum reliability based on a single slip surface means that only one mode of potential failure has been considered, but in reality the slope stability problem can be considered as a system with many possible failure modes corresponding to multiple potential slip surfaces. System reliability considers the slope stability problem in terms of a system of many potential slip surfaces. The system reliability or the system probability of failure must be estimated for comparison with the corresponding reliability or probability of failure with respect to a 'critical slip surface'. The overall probability of failure of a slope with many potential slip surfaces can be greater than the probability of failure considering any individual slip surface.

The system reliability may be considered as simple series system (failure occurs if any element of the system fails) or a parallel system (the failure of one element of the system leads to further loading of other elements and consequent decrease in reliability but the system does not fail unless all elements fail) or a combination of both. In the present study the system reliability is expressed using series system unimodal bounds i.e. first order terms are considered.

Case Study of a Finite Slope

The same Mussoorie landslide is considered again. The lower and upper bounds of the system reliability for fully independent failure modes are given in Table 20. Here also three different correlations i.e. ρ (c, ϕ) = 0.0, 0.25

TABLE 20 : System Reliability Analysis (for Statistically Independent Failure Modes)

Correlation	$(P_f)_{overall}$	System failure probability
ρ (c, ϕ) = 0.0	0.7556	$0.9894 \le P_F \le 0.9999$
ρ (c, ϕ) = −0.50	0.7808	$0.9944 \le P_F \le 0.9999$
ρ (c, ϕ) = 0.25	0.7421	$0.9864 \le P_F \le 0.9999$

and −0.50 between strength parameters are used. For this case the overall probability of failure of the slope by RPEM is 0.7556, which is very high. The failure probability of minimum reliability index will be the system reliability for the case of dependent failure modes and is more than 0.98 in this case, which is very high.

From the results of the above studies, it may be observed that, β_{min} and the corresponding surface are different from β_{det} and the critical deterministic surface. The value of β_{det} is comparable to the value of $\beta_{overall}$. However, the probability of failure considering the system reliability is higher. Hence, evaluation of system reliability is important. The upper bound of the system failure probability comes out to be 0.9999, which means the probability of safety is almost zero. The difference between upper and lower bounds is also narrow. This could be an explanation for the cause of the failure.

FIGURE 17 : A Typical Rock Slope in the Western Ghats

TABLE 21 : Dip and Dip Direction of the Joint Sets

Joint	Dip direction (degrees)	Dip angle (degrees)
J1	260	85
J2	300	55
J3	270	60
J4	290	40

TABLE 22 : Material Properties of the Rock slope

Properties	Mean	COV (%)
c (kN/m^2)	160	20
ϕ (Deg)	45	20
γ (kN/m^2)	27	5

Case Study of a Rock Slope

The concept of system reliability is now extended to planar failure along multiple joint sets. This case study refers to the failure of a cut-slope in rock at a site in the Western Ghats close to Mumbai (Fig.17). The basaltic rock in this region is always intersected by multiple joint sets. Hence system reliability analysis is warranted to evaluate the combined effect. The slope is approximately 40 m high and is intersected by multiple joint sets forming wedges. Slope and joints data is given in Tables 21 and 22. A stereoplot of the joint sets with slope face superimposed is shown in Fig.18. The effect of multiple joint sets is considered in the form of system reliability.

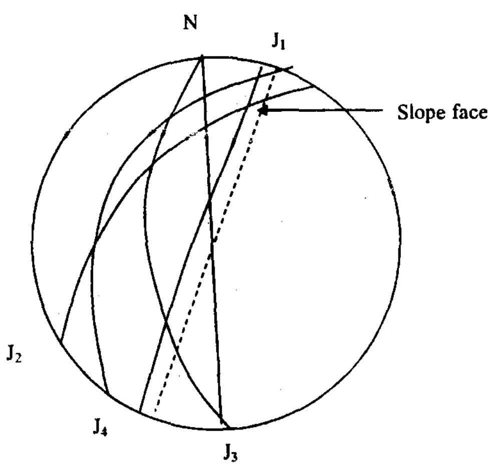


FIGURE 18 : Stereoplot of the Rock Slope With Four Joint Sets

TABLE 23 : Probability of Failure (Plane Failure)

Combination of joints	2 joints	3 joints	4 joints
Global probability of failure	0.649	0.604	0.508
System reliability	−0.380	−0.360	−0.020

Probabilities of Recurrence

The probabilities of failure and corresponding system reliability indices are given in Table 23. In addition, using the concepts of exceedence probabilities, the probabilities of their recurrence (Genevois and Romeo, 2003) are computed for a range of reported rainfall intensities and earthquake intensities typical of Western India, for a 10-year period, giving due importance to the uncertainties in the strength properties. The results are presented in Figs.19 and 20.

It can be seen that, if the excavated face of the slope had been oriented differently, the probability of failure could have been lesser. Thus, a prior analysis of the reliability could have yielded useful information regarding the appropriate orientation and dip to be given to the slope face to keep the probability of failure least. The cumulative probability of recurrence of failure considering the recurrence probability of the main triggering factor, viz., rainfall of different intensities is a measure of the risk faced by this location with time.

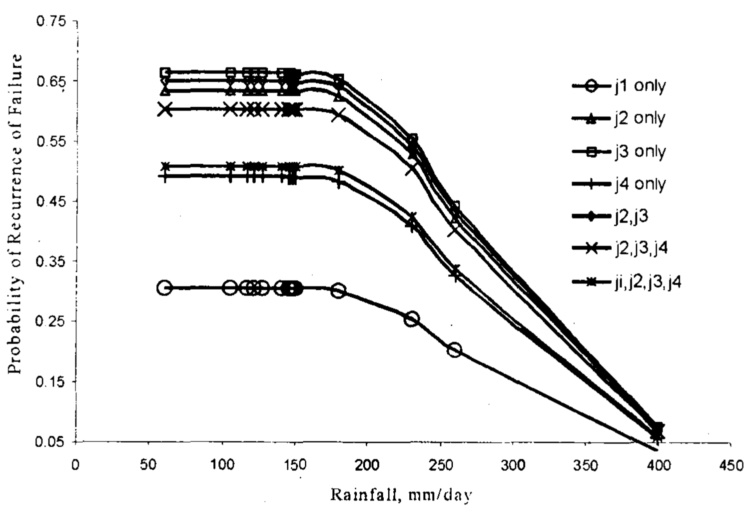


FIGURE 19 : Variation of Probability of Recurrence of Plane with Rainfall Intensity

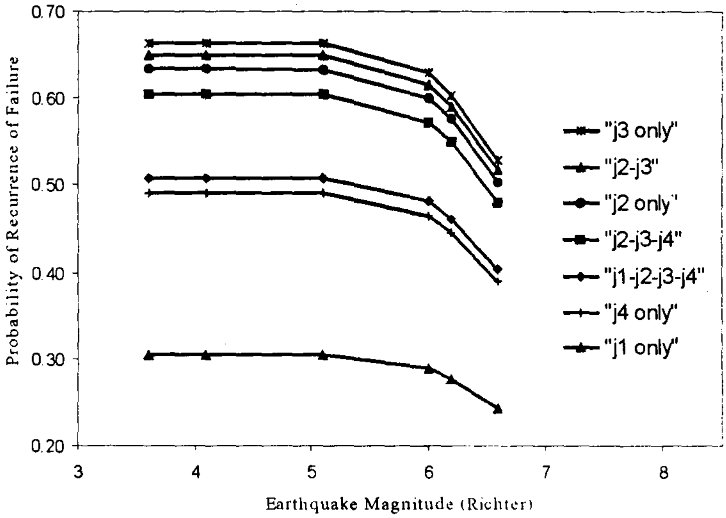


FIGURE 20 : Variation of Probability of Recurrence of Plane Failure with Earthquake Magnitude

Application to Reinforced Earth Slopes and Walls

In the previous sections natural slopes were considered. Now, the application to a typical urban slope is considered. In urban settings, limitations on space dictate the use of reinforced earth walls as retaining structures. Considerable engineering advantage (economy and performance) can be achieved by applying reliability analysis in their design.

Centrifuge Model Studies

For illustration of the application of reliability concepts to real geotextile reinforced earth slopes, centrifuge model studies (Porbaha, 1998) have been used. Since centrifuge model studies offer an excellent opportunity to simulate prototype behavior faithfully, they serve as a viable alternative to constructing an actual prototype for validation of the concepts. The model slope is shown in Fig.21. This slope was tested at different g-values until failure by varying the angle of the slope and the length of the geotextile reinforcement. Analysis of a typical case of a 1H : 3V slope with reinforcement length equal to 0.75 times the height of the slope is presented here.

Even in the model prepared under controlled conditions the values of soil properties have been reported to vary over a wide range. Therefore, the mean and COV of the material properties have been chosen as given in Table 24. Variability in properties of the geotextile (Tensile strength of model geotextile = 0.053 kN/m) is ignored.

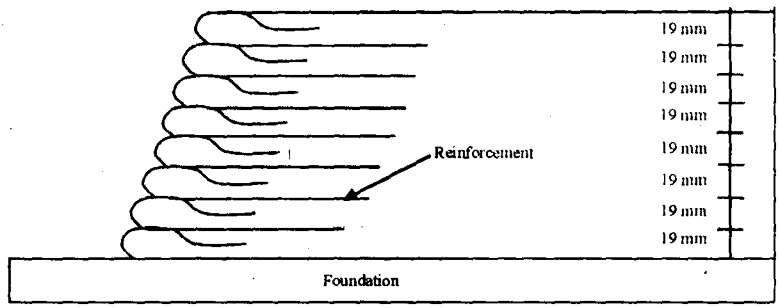


FIGURE 21 : Profile of a Typical Centrifuge Model (after Porbaha, 1998)

TABLE 24 : Material Properties of the Embankment fill for Reliability Analysis

Property	Mean	COV (%)		
		Low	Medium	High
Cohesion (kN/m^2)	20.5	10	25	40
ϕ (deg.)	20	5	15	25
γ (kN/m^3)	17	3	4	5

Both deterministic and reliability analyses have been carried out on prototype slope configurations corresponding six different g-values upto the failure-g. The values of β_{det} and corresponding p_f have been computed for the three different variabilites indicated. The results are presented in Table 25 and Fig.22.

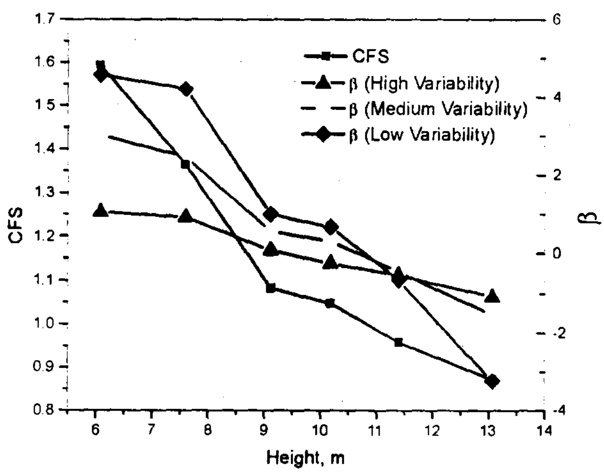


FIGURE 22a : Variation of CFS and β with Height

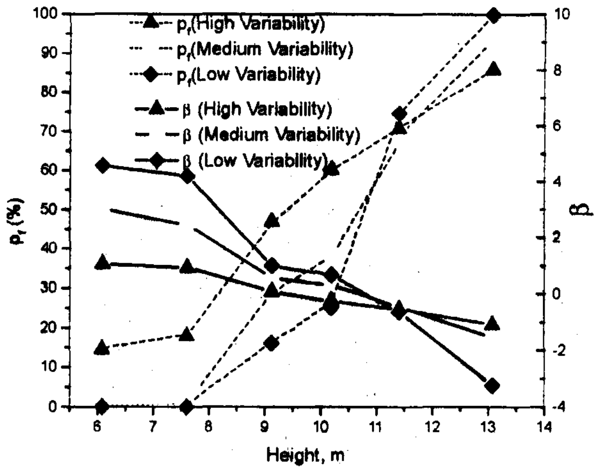


FIGURE 22b : Variation of β and p_f with Height

TABLE 25 : Results of the Reliability Analysis

a) Low Variability of Soil Properties

g level	H (m)	T_s (kN/m)	FS (CDS)	Mean FS	Sigma FS	Beta	p_f (%)
40	6.08	2.12	1.593	1.5800	0.127	4.566929	0.000248
50	7.6	2.65	1.363	1.3619	0.086	4.20814	0.001288
60	9.12	3.18	1.082	1.0783	0.079	0.991139	16.08088
67	10.184	3.551	1.047	1.0441	0.066	0.668182	25.20087
75	11.4	3.975	0.959	0.9610	0.059	-0.66102	74.56993
86	13.072	4.558	0.872	0.8509	0.046	-3.2413	99.9405

b) Medium Variability of Soil Properties

g level	H (m)	T_s (kN/m)	FS (CDS)	Mean FS	Sigma FS	Beta	p_f (%)
40	6.08	2.12	1.593	1.5749	0.190	3.025789	0.123999
50	7.6	2.65	1.363	1.3548	0.145	2.446897	0.720461
60	9.12	3.18	1.082	1.0684	0.122	0.560656	28.75161
67	10.184	3.551	1.047	1.0350	0.113	0.309735	37.83815
75	11.4	3.975	0.959	0.9538	0.100	-0.46240	67.81027
86	13.072	4.558	0.872	0.8568	0.093	-1.53978	93.81936

TABLE 25 : Contd ...

c) High Variability of Soil Properties

g level	H (m)	T_s (kN/m)	FS (CDS)	Mean FS	Sigma FS	Beta	p_f (%)
40	6.08	2.12	1.593	1.4504	0.427	1.054801	14.57582
50	7.6	2.65	1.363	1.2960	0.323	0.916409	17.97263
60	9.12	3.18	1.082	1.0207	0.27	0.076667	46.94443
67	10.184	3.551	1.047	0.939	0.237	-0.25738	60.15588
75	11.4	3.975	0.959	0.8832	0.215	-0.54326	70.65232
86	13.072	4.558	0.872	0.7917	0.194	-1.07371	85.85239

Analysis of Model Response

From the results it can be seen that, for even a slope height of 7.6 m (50 g), which is approximately half the failure height (86 g), the slope is unstable (β = 0.916 for high variability) though a CFS of 1.363 is achieved. There is a sharp decrease in β for slope heights from 7.6 m (50 g) to 9.12 m (60 g), as indicated by its value falling from 0.916 sharply to 0.077. A similar trend in decrease in β is observed at low variability too. This probably indicates deterioration of the slope and correlates well with the appearance of crack at 10.18 m (67 g).

This calls for the need to design such slopes based on β rather than on deterministic concepts. In the following section, a simplified approach to reliability based design of a geotextile reinforced earth wall is presented.

Simplified Reliability Based Design for Internal Stability

Geotextile reinforced earth walls are now preferred for retaining urban slopes due to constraints on availability of space. The concepts of reliability now permit their safe and economical design for a given probability of failure or reliability index rather than for a given factor of safety.

Consider a geotextile reinforced earth wall as shown in Fig.22. For a given soil type and wall height H, the design mainly involves the determination of the required tensile strength, length and spacing of the geotextile reinforcement. Key governing parameters are the unit weight of the backfill soil (γ), the angle of internal friction (ϕ), the angle of sliding resistance between the soil and the reinforcement (ϕ_u) and the allowable tensile strength of the reinforcement (T_{all}). A simple approach which can be easily implemented in practice is used as explained below:

The required tensile strength and length of the reinforcement are now taken as random variables and that their variabilities are affected by the variabilities of the backfill soil parameters γ and ϕ, in the sense, that, at limit equilibrium, they have to satisfy internal stability. Simple relationships are now derived for the mean and variance of the reinforcement properties in terms of the known variability of soil properties and the probability distributions of required tensile strength (T_S) and length are calculated. This helps to develop design aids.

The probability of failure of the retaining wall and the reliability of the structure for a given situation can be evaluated if the probability distribution of the provided tensile strength (T_R) is known. Alternatively, the tensile strength required for a given probability of failure can be calculated.

The first step is to obtain expressions for the expected value and variance of the critical parameters T_S and L. Considering active earth pressure conditions in the backfill, the following equations can be written:

$$S_V \sigma_h = 2\tau L_e = T_S < T_{all} \tag{44}$$

$$L = L_L + L_e \tag{45}$$

$$L_L = (H - z)\tan\left(45 - \frac{\phi}{2}\right) \tag{46}$$

$$L_e = \frac{S_V \sigma_h}{2(c + \gamma z \tan\phi_u)} \tag{47}$$

where

S_V = lift thickness (vertical spacing)

σ_h = soil pressure at the depth considered.

τ = resistance offered by soil to geotextile

T_{all} = allowable tensile strength of geotextile

L = total length of geotextile

L_L = distance from face to failure surface

L_e = the required embedment length

c = cohesion (zero, if granular backfill is used)

z = depth from ground surface

ϕ_u = the angle of shearing resistance between soil and the geotextile (usually 0.67ϕ to 0.90ϕ)

Mean and Variance of T_s

For a given spacing S_V, the expected value of tensile strength T_S may be written based on Eqn.44 as

$$E(T_S) = E(K_a)\,E(\gamma)\,zS_V \tag{48}$$

where $E(K_a)$ and $E(\gamma)$ are the expected values for K_a and γ respectively.

Similarly, variance of T_S is given by

$$\mathrm{Var}(T_S) = \left(\frac{\partial T_S}{\partial K_a}\right)^2 \mathrm{Var}(K_a) + \left(\frac{\partial T_S}{\partial \gamma}\right)^2 \mathrm{Var}(\gamma) \tag{49}$$

The variance of γ is given by

$$\mathrm{Var}(\gamma) = \left\{\mathrm{CoV}_\gamma \cdot E(\gamma)\right\}^2 \tag{50}$$

Similarly, Variance of K_a is given by

$$\mathrm{Var}(K_a) = \left(\frac{\partial K_a}{\partial \phi}\right)^2 \mathrm{Var}(\phi) \quad \text{and} \tag{51}$$

$$\mathrm{Var}(\phi) = \left\{\mathrm{CoV}_\phi \cdot E(\phi)\right\}^2 \tag{52}$$

Hence for a given value of z, γ, CoV_γ , ϕ , CoV_ϕ, S_V and wall height the expected value and variance of T_S can be calculated.

Mean and Variance of Reinforcement Length

The reinforcement length L is the summation of L_L and L_e. The equations for approximate mean and variance for each part of the length are:.

From Eqn.3, the expected length and variance of L_L is given by

$$E(L_L) = H\left[1 - \frac{z}{H}\right]\tan\left[45 - \frac{E(\phi)}{2}\right] \tag{53}$$

$$\mathrm{Var}(L_L) = \left(\frac{\partial L_L}{\partial \phi}\right)^2 \mathrm{Var}(\phi) \tag{54}$$

Similarly,

$$L_e = \frac{S_V \sigma_h}{2\gamma z \tan\phi_u} = \frac{S_V K_a \gamma z}{2\gamma z \tan\phi_u} = \frac{S_V K_a}{2\tan\phi_u} \tag{55}$$

Since ϕ_u is usually expressed as a fraction of ϕ, $(\delta = \phi_u/\phi)$, L_e is dependent only on the statistics of ϕ for a given S_V. Therefore the expected value and variance of L_e is given by

$$E(L_e) = \frac{S_V E(K_a)}{2\tan\{\delta \cdot E(\phi)\}} \tag{56}$$

$$\mathrm{Var}(L_e) = \left(\frac{\partial L_e}{\partial K_a}\right)^2 \mathrm{Var}(K_a) + \left(\frac{\partial L_e}{\partial \phi_u}\right)^2 \mathrm{Var}(\phi_u) \tag{57}$$

Finally, assuming L_L and L_e to be statistically independent,

$$E(L) = E(L_L) + E(L_R) \text{ and} \tag{58}$$

$$\mathrm{Var}(L) \cong \mathrm{Var}(L_L) + \mathrm{Var}(L_e) \tag{59}$$

Development of Design Aids

System reliability is considered. It can be seen that ϕ, γ, COV_ϕ and COV_γ play an important role. Loose, medium and dense sands are represented in terms of ϕ = 30°, 35° and 40°.

The range of densities for these sands are assumed to be (15.7 - 18.9), (17.3 - 20.5) and (18.9 - 22.0) kN/m^3 respectively. Values of (COV_γ) = 0.10, 0.20 and 0.30, respectively and values of (COV_ϕ) = 0.2, 0.3 and 0.4 are used. The height of the wall is assumed to vary between 3 - 15 m for purposes of illustration (after Basheer and Najjar, 1996).

Consider Fig.22. Let there be N number of layers of reinforcement. It is possible to develop a series of charts to carry out design to satisfy a given reliability index.

The failure mode for the layers in the upper 0.4 H is assumed to be rupture or pullout, while that for the lower 0.6 H is purely rupture. The minimum length of reinforcement to be provided is 0.7 H. If N_1 and N_2 are the number of layers of reinforcement in the top and bottom zone, then for a required system reliability of R_S, the critical length L^* of the reinforcement may be written as

$$P\left[\left(L_i \le L_i^*\right),\left(T_{sj} \le T_{Rj}\right)\right] = R_S\,,\ \text{where } i = 1,\ N_2 \text{ and } j = 1,\ N \tag{60}$$

$$P\left(L_i \le L_i^*\right) = R_S^{\{1/(N_2+N)\}} \tag{61}$$

$$P\left(T_{Sj} \le T_{Rj}\right) = R_S^{\{1/(N_2+N)\}} \tag{62}$$

Using the standard normal distribution tables the critical reinforcement length L_i^* is given by

$$\begin{aligned} L_i^* &= E\left(L_i\right) + \xi\sigma_{L_i} \\ &\ge 0.7H \end{aligned} \tag{63}$$

Now, typical design aids such as those in Fig.23 can be prepared for different types of sands for a vertical spacing of reinforcement as 1 m, and δ = 0.67. The z/H values corresponding to L^*/H of 0.7 are marked on these graphs. The maximum value of z/H is 0.4 for a reliability index of 3 with COV_ϕ of 0.5. Usually the reliability indices used in geotechnical engineering is of the order of 3 and hence the probability of pullout failure for the layers above a z/H of 0.4 is assumed as zero and hence marked as tension failure zone.

To illustrate the usage of risk based design approach, let us use the values of $E(T_S)$ = 18.7 kN/m and $Var(T_S)$ = 59.8 $(kN/m)^2$ obtained for a wall of 9 m height at a z/H of 0.8 with COV_ϕ = 0.3 and COV_γ = 0.2.

Assuming that the available reinforcement material is having COV_{TR} = 0.10, the CSF required for the layer at z/H = 0.8 is 2.19. The required mean geotextile (or reinforcement) strength at this level should be more than 2.19 × 18.7 = 40.95 kN/m and the length should be 0.7 H = 6.3 m as this layer is in tension failure zone.

To calculate the length required for the top most layer, at z = 0.9 m, assume δ = 0.67.

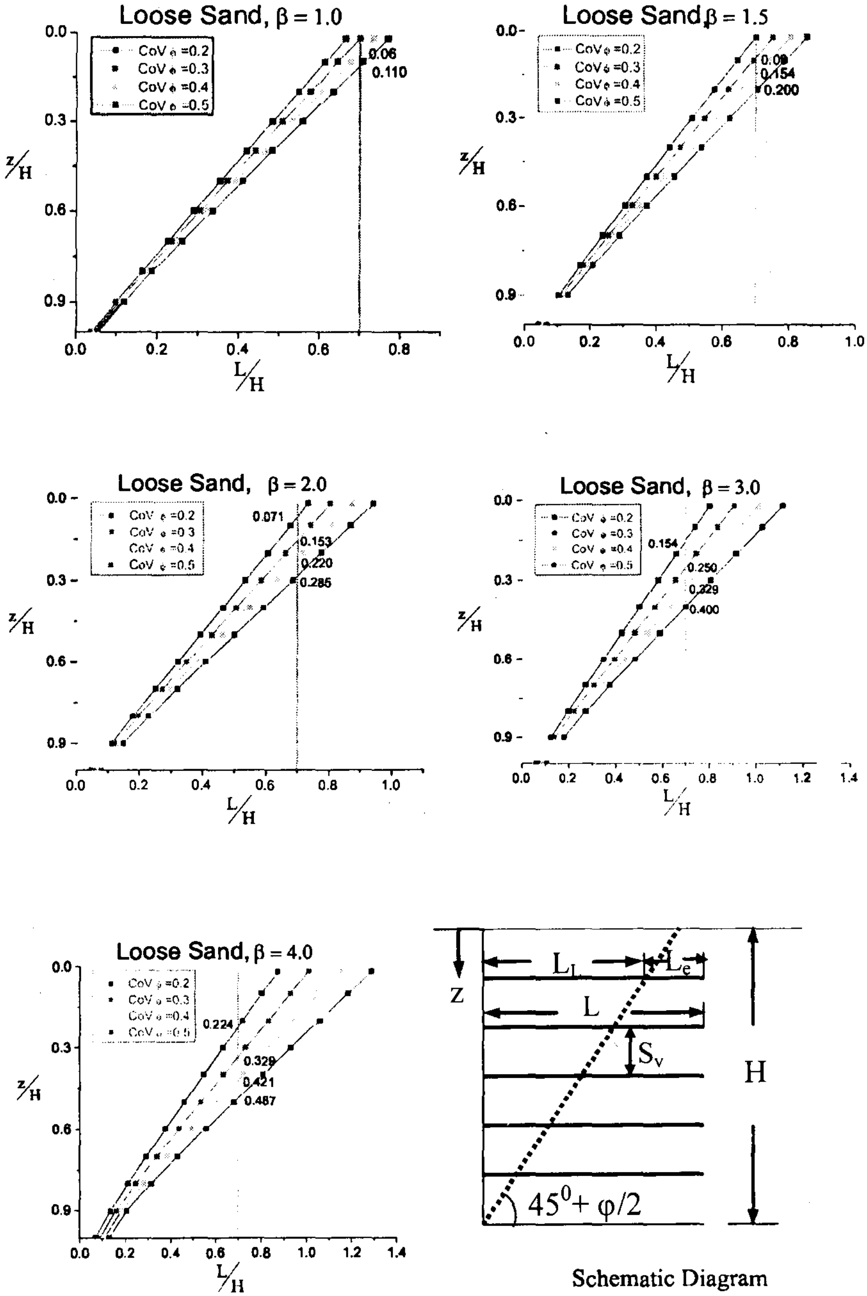


FIGURE 23 : Plots showing the required L/H for different z/H and β for Loose Sand (δ = 0.67, S_V = 1.0 m)

$$E(L_L) = 5.2\ (1-0.1) = 4.68\ \text{m}$$

$$Var(L_L) = 0.67 \times 1\ (1-0.1^2) = 0.6633\ \text{m}^2$$

$$E(L_e) = 0.455 \times 0.9 = 0.41\ \text{m}$$

$$Var(L_e) = 0.05 \times 1 \times 081 = 0.04\ \text{m}^2$$

$$E(L) = E_L + E_{L_e} = 5.09\ \text{m},$$

$$Var(L) = 0.7038\ \text{m}^2$$

$$\text{Therefore, } L^* \geq 5.09 + 2.54 \times (0.7038)^{0.5} = 7.22\ \text{m}$$

This procedure should be followed at each value considered for placing a geotextile for the complete design of the wall.

Study of Regional Hazard Variation in GIS Environment

On a regional scale the study of slope instability takes the shape of hazard zonation, i.e. delineating zones of different degrees of hazard from slope instability point of view. The availability of GIS softwares makes it now possible to carry out such studies easily. A map showing spatial distribution of hazard classes is known as a Hazard map. This is particularly suited for studying natural slopes or landslides. It provides useful information about the relative hazard of the units constituting the study region. In fact, there is a whole new field of activity known as integrated hill area development, for which this is a crucial input.

Qualitative and Semi-Quantitative Methods

Hazard zonation techniques revolve around the concept of relative ranking of the basic land units based on their perceived proneness to landslide occurrence. The ranking techniques used are necessarily qualitative or semi-quantitative owing to the difficulties of data collection over large areas.

Hazard mapping has been carried out for several regions in the country by different agencies. The procedures adopted in these studies vary in detail, but, in general, the underlying concept is the same. It involves identifying a set of factors – geological, geomorphological, hydrological and botanical – which have a bearing on slope instability. The typical parameters considered are given in Table 26. Then their relative importance is estimated on the basis of past experience, assigned a 'weightage' and the area is divided into zones of different cumulative weightages reflecting a hazard intensity. The basic terrain unit, which is considered for computing intensity, may be a land

TABLE 26 : Typical Parameters for Hazard Zonation

S. No.	Parameter
1	geology (lithology)
2	structure, lineaments
3	landform, geomorphology
4	soil type, soil thickness
5	slope
6	relief
7	vegetation
8	land use – land cover
9	groundwater conditions
10	drainage pattern
11	drainage density
12	rainfall
13	landslide history
14	other site specific parameters

facet or a grid cell. The importance of weightages and need for alternative approaches has been highlighted by many.

Similarly, the scales of the input source maps or remotely sensed data and their quality also have a bearing on the zonation. Aerial photographs especially of higher scales like 1 : 20,000 or more are better for the landslide study. The use of scales of 1 : 5000 to 1 : 10000 for areas of a few tens of square meters and 1 : 100000 for areas of the order of 2500 square meters is common.

Topographic sheets of scale 1 : 50000 have been common in many studies. RADARSAT data holds good promise, since it is not affected by clouds and some of the limitations of other remotely sensed data are overcome.

Broadly, the methods used may be categorized into supervised and unsupervised, the difference being that the former use 'a priori' knowledge of past occurrences of landslides in the region. The latter is necessary when the study area is inaccessible or across the borders. However, with remote sensing possible, some 'a priori' information can be obtained relatively easily.

In both the methods, the expert first identifies the factors contributing to slope instability and gives relative weightages to them based on his accumulated experience and judgment of their relative importance. The scales used for assigning weightages vary anywhere from 10 to 100.

Next, factor maps, showing the ranges of variations of these factors in the study area, are prepared. Remote Sensing is a very useful tool for this. The ranges are divided into sub-categories and assigned ratings as a proportion of the weightage given to that particular factor. At this stage the supervised methods use 'a priori' knowledge. There is a class of supervised methods (regression analysis, Information Value method, ANN or GA method) in which the ratings are computed rather than assigned thereby introducing a rigidity and leaving little scope for informed judgment.

Next, the study area is divided into units (varying from land facets to regular grids) and the cumulative rating of all the sub-categories of all the factors within each unit is computed. GIS is an useful tool for computing the cumulative rating. This cumulative rating is a measure of the hazard.

Lastly, the study area is divided into zones of different hazard levels and hazard zonation is complete.

The major drawback in these approaches is the subjectivity associated with the assignment of weightages and ratings. Therefore, a probabilistic method based on the RPEM is evolved and recommended. This method accounts for the subjectivity by accommodating variability in the weightages.

The illustration presented here uses the GIS software GRAM++ developed by CSRE, IIT Bombay. It is based on raster analysis, which is found to be more suitable for landslide hazard zonation as arithmetic operations can be done at a pixel level, which tend to improve the accuracy of predicted hazard zone than conventional grid wise calculations.

Four methods of hazard zonation were applied to the same area including the proposed probabilistic approach to prepare Landslide Hazard Zonation maps and to find out the degree of reliability of each method. The methods used for the analysis are:

Landslide Hazard Evaluation Factor (LHEF) method,
Landslide Susceptibility Value (LSV) method,
Information Value (IV) method and
The proposed probabilistic method based on RPEM.

Subsequently, Distribution Ratio method is used to check the confidence of the predicted results. These methods of analysis and the results of hazard zonation are presented in the following sections.

Study Area

The area chosen for the present study of landslide hazard assessment comprises 750 km^2 falling within latitude (9° 30′) and (9° 45′) and longitude (76° 45′) and (77°). Administratively the area falls within Idukki and Kottayam districts of Kerala. The area forms part of the highland region of Kerala. The area has all the physiographic components of a typical vulnerable zone of Western Ghats. This area has experienced multiple slides in the recent past.

The major parameters considered for the study are: Geology, Slope, Soil thickness, Land use, Relative relief, Drainage pattern and Drainage density.

Thematic maps for the above parameters were available from Centre for Earth Science Studies, Thiruvananthapuram, (Thampi et al., 1998), prepared using remotely sensed data, toposheets, other maps and ground truths. The maps are rasterised by dividing each map into 398790 (= 633 × 630) pixels. The basic raster maps used for the present work are shown below (Figs.24 to 30).

The issues associated with hazard mapping are rooted in the method adopted for assigning weightages to the factors in each region. It is interesting to note the vast variations in the weightages in the various methods in vogue. Some typical weightages assigned in studies in two different settings are listed below in Tables 27 and 28.

In the first variation, the parameters are assigned a LHEF value on a scale of 10 (Anbalagan, 1992). These weighted factor maps are then summed up using the arithmetic operator to get the Total Estimated Hazard (TEHD) map. Then the study area is divided into different landslide hazard zones based on the TEHD values.

In the second, the parameters are assigned LSVs on a scale of 100. Then LSI values for each sub parameter are calculated using the following equation (Thampi et al., 1998). Hazard zones are based on cumulative LSIs of all contributing factors.

$$\mathrm{LSI} = \frac{(\mathrm{Landslide/sq.km.})\,\% * \mathrm{LSV}}{100} \tag{64}$$

The basic principle involved in the Information Value method is to relate the weightages to the known occurrences of landslides. It considers two ratios for the calculation of weightages of each category of a parameter.

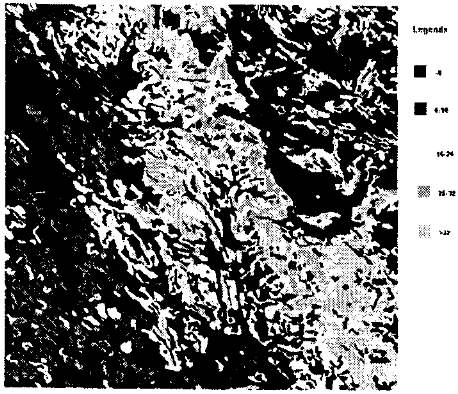


FIGURE 24 : Slope Map

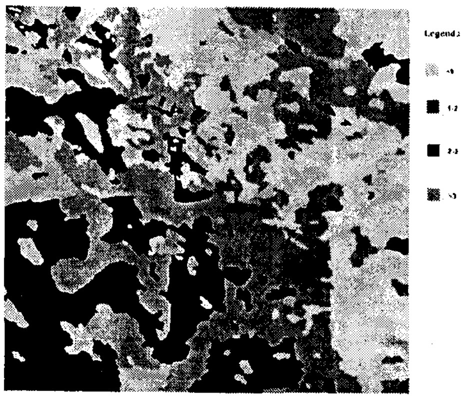


FIGURE 25 : Soil Thickness Map

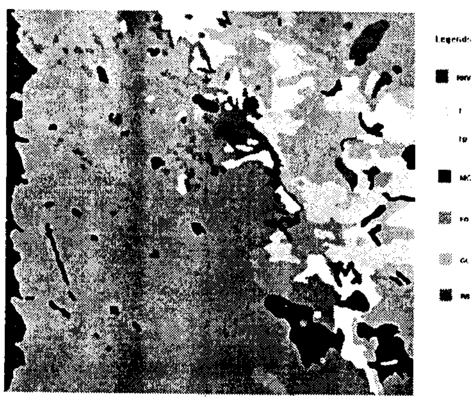


FIGURE 26 : Land use Map

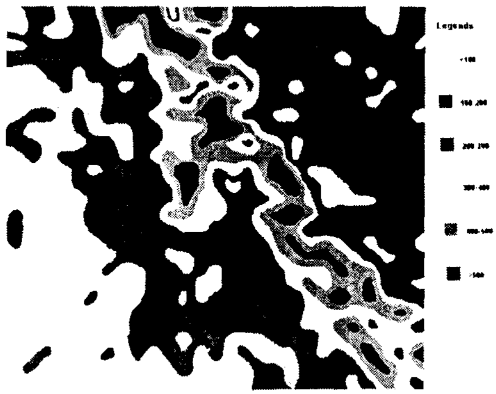


FIGURE 27 : Relative Relief Map

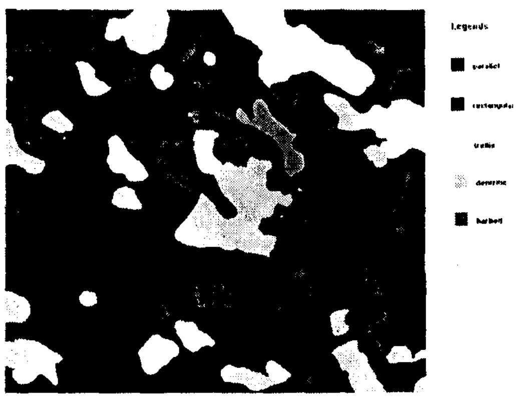


FIGURE 28 : Drainage Pattern Map

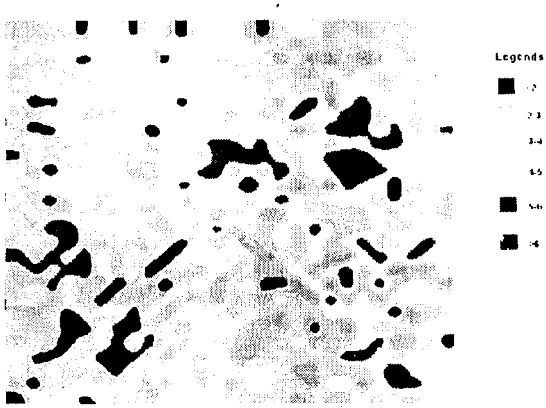


FIGURE 29 : Drainage Density Map

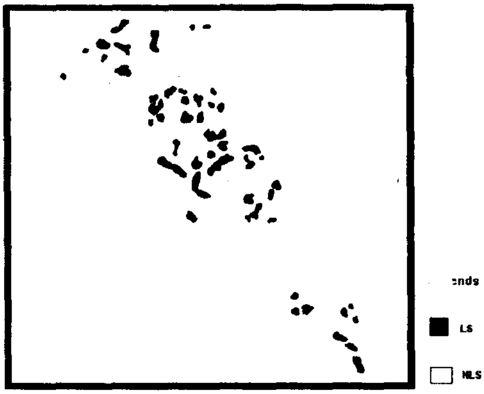


FIGURE 30 : Landslide Distribution Map

TABLE 27 : Setting 1: Garhwal Himalayas

S.No.	Contributory Factor	Maximum LHEF* Rating
1.	Lithology	2
2.	Structure	2
3.	Slope Morphometry	2
4.	Relative Relief	1
5.	Lan duse Land cover	2
6.	Groundwater condition	1
	Total	10

* Landslide Hazard Evaluation Factor (After Anabalagan, 1992)

TABLE 28 : Setting 2: Western Ghats – Kerala

S.No.	Contributory Factor	LSV*
1.	Slope	30
2.	Soil Thickness	25
3.	Landuse	15
4.	Relative Relief	10
5.	Drainage Pattern	10
6.	Drainage Density	05
7.	Landform	05
	Total	100

* Landslide Susceptibility Value (After Thampi et al., 1998)

First is the ratio of number of elements with history of landslide occurrence involving variable or category I (S_i) to the total number of elements of a particular category of a parameter (N_i) and second is the ratio of total number of elements having history of landslide occurrences including all the categories of a parameter (S) to the total number of elements in the entire area (N). The weightages to each category known as Information Value is calculated from the above two ratios using the equation given below.

$$I_i = \log \frac{S_i/N_i}{S/N} \tag{65}$$

where I_i = Information value provided by variable or category i for landslide to happen

Suggested Probabilistic Method

In all the methods for Hazard Zonation, landslide hazard map is primarily dependent on the weightages given to the different contributing factors. In the first two methods, LHEF method and LSV method, weights are given on the basis of the experience by the analyzer, which is very much predisposed to subjectivity. Hence the weightages given will vary in certain range. So in order to see the effect of variation in the major parameters and to reduce the subjectivity in assigning weightages to a certain extent a probabilistic method based on RPEM is evolved and applied on the LSV method of landslide hazard zonation. RPEM not only assumes a limiting equilibrium condition but also takes into account the statistical distribution of the input parameters. In the present study the coefficients of variations assumed for the major parameters based on the random nature of each parameter are given in Table 29.

The LSI maps prepared for the major parameters in LSV method are used as the basic maps for RPEM. In this method (Sabu, 2005), instead of

TABLE 29 : Coefficient of Variation

S. No.	Parameters	COV
1	Slope	20%
2	Soil Thickness	20%
3	Land Use	15%
4	Relative Relief	10%
5	Drainage Pattern	5%
6	Drainage Density	20%

taking the sum of mean LSIs of all the parameters, the hazard zonation is done based on the Landslide Susceptibility Factor (α) calculated using the following formula:

$$\text{Landslide Susceptibility Factor } (\alpha) = \frac{E\left[LSI_{CU}\right] - LSI_{CR}}{\sigma\left[LSI_{CU}\right]} \tag{66}$$

where LSI_{CR} = the critical value of cumulative LSI value that divides the stable and unstable zones. (In the present study the upper limit of cumulative LSI value for moderately stable zone in LSV method, which is equal to 15, is taken as the LSI_{CR} value.)

$E\,[LSI_{CU}]$ = the first expectation of cumulative LSI values which is equal to the mean value

$\sigma\,[LSI_{CU}]$ = the standard deviation of the cumulative LSI values in each grid/pixel and is obtained by taking the square root of variance ($V\,[LSI_{CU}]$)

The variance map is prepared from the $E\,[LSI_{CU}^2]$ map and $E\,[LSI_{CU}]$ map using the following formula:

$$V\left[LSI_{CU}\right] = E\left[LSI_{CU}^2\right] - \left(E\left[LSI_{CU}\right]\right)^2 \tag{67}$$

where $E\,[LSI_{CU}^2]$ is the second expectation of cumulative LSI values.

Confidence Analysis Using Distribution Ratio Method

The classification of landslide hazard zones in all the methods is the result of repetitive overlay of the thematic maps of inferred instability and the inventory of past landslides. However, a certain number of landslides still fall in the areas categorized as safe slopes or stable zones. Therefore, in order to measure the degree of reliability of the predicted results a confidence analysis is performed using Distribution Ratio method (Weerasinghe, 1998). Percentage Distribution Ratios (% DR) are computed as given below using the information regarding the area of each category and area of landslides in each category.

$$\text{DR for Hazard Zone 1} = \frac{\left(\%\, L_s \text{ Area for Hazard Zone 1}\right)}{\left(\% \text{ Area for Hazard Zone 1}\right)}$$

$$\% \text{ DR for Hazard Zone } 1 = \frac{\text{DR for Hazard Zone 1}}{\sum_{1}^{N} \text{DR for each Hazard Zone}}$$

where n = number of hazard zones

To generate the data required for the analysis, the output maps (i.e. the final landslide hazard zonation maps) of each method is superimposed with the existing landslide distribution map (Fig.31). The areas corresponding to each category and their landslide affected areas are calculated. These areas are used for the calculation of Distribution Ratio.

Comparative Evaluation

In the Distribution Ratio method the confidence of predicting landslide hazard is analyzed based on the percentage Distribution Ratios of various hazard zones and, in particular, the Critical and Highly Unstable zones. The percentage distribution ratio for different methods of hazard zonation are tabulated in Table 30.

The Information Value Method and the Probabilistic Method score the highest confidence values. However, in the Information Value method, there

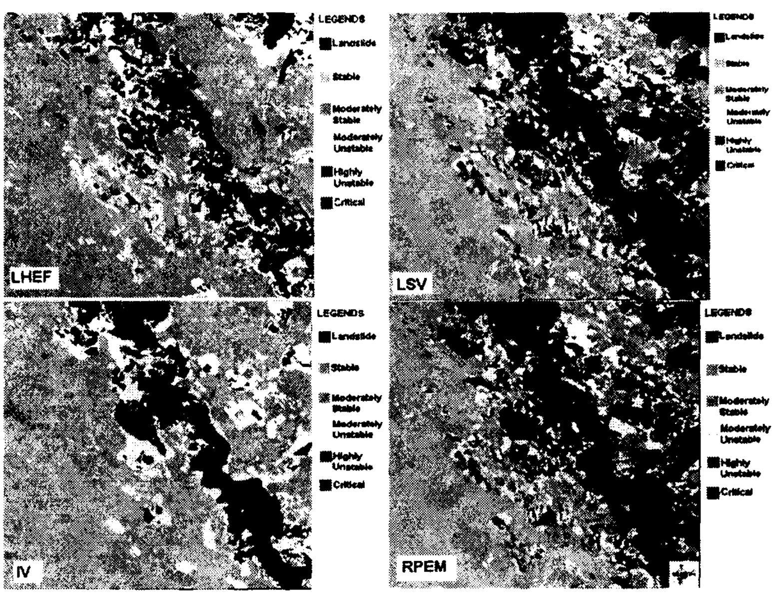


FIGURE 31 : A Comparison of the Four Zonation Methodologies

TABLE 30 : Comparison of Percentage Distribution Ratios

S.No	Hazard Zone	Percentage D.R.			
		LHEF Method	LSV Method	Information Value Method	RPEM Method
1	Stable	0	1.05	0.31	1.17
2	Moderately stable	3.39	10.54	1.93	9.53
3	Moderately unstable	31.78	11.78	15.27	8.53
4	Highly unstable	27.12	23.55	33.20	21.07
5	Critical	37.71	53.08	49.29	59.70
Confidence Value		64.83	76.63	82.49	80.77

is an inherent rigidity due to the parameter weightages being computed rigorously through regression from known occurrences of landslides, rather than being assigned through robust judgement. This makes the Probabilistic Method a preferable method. Although the LHEF method scores only 64.83%, it could be very useful for inaccessible areas, where no 'a priori' information would be available for implementing the supervised methods. The present studies are restricted to random uncertainties although in reality the uncertainties associated with 'assignment' of weightages is largely fuzzy and the fuzzy methods suggested herein would be more appropriate.

Concluding Remarks

Uncertainties are unavoidable at any stage in a geotechnical engineering activity. A conservative design using safe values of parameters may be, more often than not, an inappropriate solution involving an unknown amount of risk. Since most uncertainties in geotechnical engineering are of an epistemic nature, engineering challenge lies in recognizing them and modelling them using reliability analysis. Simple solutions have been suggested here for handling random and fuzzy uncertainties, their spatial and temporal variations. Applications to natural and man-made urban slopes and regional hazard evaluation have been presented.

Reliability analysis is all about the confidence that can be reposed on a deterministic solution. Validation is beset with difficulties, since it is difficult to perceive or measure the level of confidence directly. Centrifuge modelling appears to be a good viable solution for validating reliability based designs.

Cognitive uncertainties could offset an otherwise good solution even in engineered slopes. Therefore, there is a need for wider use of principles of probability, reliability, safety and risk. There is a need for evolving more guidelines too to define an acceptable reliability index under a variety of common situations, commensurate with the acceptable levels of risk. Much insight can be gained into the state of health of a slope or any other geotechnical structure from reliability analysis.

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Reliability and Risk Analysis of Slopes

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There are many uncertainties associated with parameter values used at every stage of a geotechnical activity. In particular, with reference to slopes, there are many subjective decisions to be made during the choice of design parameters. Deterministic slope stability analysis does not account for the uncertainties. Therefore, probabilistic analysis and fuzzy random analysis are required to handle the cognitive and non-cognitive uncertainties.

This paper proposes point-estimate based methods for analysis of reliability, including system reliability, spatial variation of reliability and regional hazard.

The proposed methods are applied to natural and man-made urban slopes, such as geotextile reinforced earth slopes and the need for reliability analysis is illustrated. The usefulness of centrifuge modelling is also highlighted.

KEYWORDS : Slope stability, Reliability analysis, Probabilistic methods, Fuzzy numbers.